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# How books tell a History of Statistics in Portugal: Works of foreigners, *estrangeirados*, and others

Dinis Pestana and Rui Santos

Abstract. Foreigners and "estrangeirados", an expression meaning "people going to a foreign country ["estrangeiro"] getting there further education", had a leading role in the development of Mathematical Statistics in Portugal. In what concerns Statistics, "estrangeirados" in the nineteenth century were mainly liberal intellectuals exiled for political reasons. From 1930 onwards, the research funding authority sent university professors abroad, and hired foreign researchers to stay in Portuguese institutions, and some of them were instrumental in the importation of new concepts and methods of inferential statistics. After 1970, there was a huge program of sending young researchers abroad for doctoral studies. At the same time, many new universities and polytechnic institutes have been created in Portugal. After that, aside from foreigners who choose to have a research career in those institutions and the "estrangeirados" who had returned and created programs of doctoral studies, others, who hadn't the opportunity of studying abroad, began to play a decisive role in the development of Statistics in Portugal. The publication of handbooks on Probability and Statistics, thesis and core papers in Portuguese scientific journals, and also of works for the layman, reveals how Statistics progressed from descriptive to a mathematical discipline used for inference in all fields of knowledge, from natural sciences to methodology of scientific research.

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Dedicated to Fernando Rosado, in gratitude for his outstanding edition of the *Boletim da Sociedade Portuguesa de Estatística* and the *Memorial da Sociedade Portuguesa de Estatística*.



"Statistics is the grammar of science." Karl Pearson

# 1 Introduction

Sketching the History of Statistics in Portugal naturally requires discussing the influence of foreigners and "*estrangeirados*", foreign-influenced individuals<sup>1</sup>, without neglecting the role of "others". Plagiating Mae West, who when asked what kind of men

<sup>&</sup>lt;sup>1</sup> In the 17th and 18th centuries, some Portuguese intellectuals, who were exposed to scientific innovations and the Enlightenment philosophy abroad, had a far-reaching influence on the modernization of culture in Portugal. Among these "*Estrangeirados*" (as they are referred to in Portuguese history) are Ribeiro Sanches, Luís António Verney, Francisco Xavier de Oliveira (Cavaleiro d'Oliveira), D. Luís da Cunha (whose political testament recommended Sebastião José de Carvalho e Melo, later the powerful prime minister Marquês de Pombal, to King D. José), and the Marquês de Pombal himself. The Pombaline reform of the university in 1772 was certainly due to the influential actions of the *Estrangeirados*.

The liberal struggles of the 19th century forced many illustrious Portuguese to exile, particularly in the United Kingdom and/or France. Some of these new *estrangeirados*, namely Solano Constâncio, Alexandre Herculano, and Oliveira Marreca, were important agents for the dissemination of Statistics in Portugal. The politician Passos Manuel, exiled in La Coruña-Plymouth-Belgium-Paris between 1828 and 1832, carried out important education reforms during his brief tenure as Minister of the Kingdom in 1836-1837, both in secondary and higher education. One of the consequences of his reform was the creation of a course of Economia Política [Political Economy], whose Professor, Adrião Forjaz de Sampaio, included the teaching of *Princípios de Statistica* [Statistical Primer] [52] in its program from 1841 onwards.

We use the term *estrangeirado* in a very general sense: *estrangeirados* will also be much later Zaluar Nunes (first as a student in Paris, then as a political exile in Brazil, due to the diaspora of intellectuals persecuted by Salazarism), Bento Murteira in London, Gustavo de Castro in Paris, Sebastião e Silva in Italy, Dias Agudo, Tiago de Oliveira, and Pedro Braumann in the USA, and later many scholarship holders who pursued doctorates abroad.

she preferred, candidly replied: "Personally, I like two types of men — domestic and foreign", in our opinion many "Others", who hadn't the opportunity of extended partnerships abroad, had also a relevant role in the development of Mathematical Statistics in Portugal, and their contributions must be recognised and praised as much as those of the estrangeirados. Notorious examples are António Simões Neto, a man of culture and one of the early enthusiasts of Bayesian ideas in Portugal, who influenced many students at the Faculty of Sciences of Lisbon (namely Maria Antónia Amaral Turkman and Dinis Pestana), Fernando Rosado (to whom we dedicate this sketch of the history of Statistics in Portugal, thanking his contribution to a "Portugaliae Monumenta Statistica" with his fine edition of the Boletim da Sociedade Portuguesa de Estatística [148] [Bulletin of the Portuguese Statistical Society]) and of the Memorial da Sociedade Portuguesa de Estatística [147] [Memorial of the Portuguese Statistical Society], Salomé Cabral or Manuela Neves (recently recipient of the Portuguese Statistical Society Life Achievement Award), whose doctoral dissertations were supervised by Tiago de Oliveira, or Rita Vasconcelos, who contributed so much for the progress of the Probability and Statistics group at the University of Madeira.

Regarding *estrangeirados*, there is quite a bit of information, but when it comes to the influence of foreigners, the gaps are significant, especially considering the enormous longdistance influence of works already mentioned in the 19th century (Quételet by Forjaz de Sampaio, Bertrand and Poincaré in Sidónio Paes's thesis) or used as recommended bibliography in courses or research seminars (for example, books by Fisher, Kendall and Stuart, Cramér, Loève, Marek Fisz, Feller, Gnedenko, Cox, Zar, Rohatgi, ...).

Information is also quite lacking about the actual role of foreigners who came to Portugal as invited congress participants, and even more so about those who came on their own initiative as participants. There is also little information on the relevance to the development of Statistics in Portugal of those who served on advisory boards of research centers.

On the other hand, it is interesting to list which foreign statisticians were awarded honorary doctorates by Portuguese universities, which foreigners chose to spend part of their academic life in Portugal, and which books were translated and served as useful aids in the teaching, research, and dissemination of Statistics in Portugal.

Furthermore, research in university libraries and the National Library allowed us to identify and consult a reasonable number of statistical works that have been published, providing a reliable indicator of how the introduction of Mathematical Statistics concepts and the evolution of their teaching and dissemination progressed.

Although we are aware that we are far from an exhaustive presentation, we will attempt to give a coordinated overview of how the teaching and dissemination of Statistics developed in the 19th century (Section 2). During this period, the increasing importance of official statistics and the introduction of Probability teaching at the Polytechnic School contributed to the recognition of Statistics as a mathematical discipline, culminating with Sidónio Paes submission of the first doctoral thesis in the field of Mathematical Statistics in Portugal, at the Faculty of Mathematics of the University of Coimbra.

Concerning the advancements in Mathematical Statistics in the 20th century up to 1974 (Section 3), we highlight the consequences of starting the teaching of Probability at the

Faculdade de Ciências [Faculty of Sciences]. Excellent professors developed the teaching of Statistics at the Instituto Superior de Agronomia (ISA) [Higher Institute of Agronomy] — Varennes e Mendonça, Zaluar Nunes, and Sebastião e Silva — and at the Instituto Superior de Economia e Gestão (ISEG) [Higher Institute of Economy and Management], namely Leite Pinto and Bento Murteira, who worked under Maurice Kendall in London. These professors imported many new statistical concepts, particularly the innovations created by K. Pearson, 'Student', Neyman and E. Pearson, Fisher, and their co-workers. Fisher's influence was significant at ISA, but we should not overlook the potential role the Institute of Anthropology in Coimbra could have played due to the hiring of Stevens, a disciple of Fisher, who published a series of remarkable "Methodological Issues" in the *Revista da Faculdade de Ciências da Universidade de Coimbra* [Review of the Faculty of Sciences of the University of Coimbra]. Finally, we include some informal notes about the last fifty years (Section 4), for which it seems premature to adopt a historical perspective.

# 2 19th Century

The Venetian geographer and statistician Adriano Balbi (1782-1848) was in Portugal in 1820, before settling in Paris from 1821 to 1832, where he changed his name to Adrien Balbi. It was in Paris, in 1822, that he published the two volumes of the *Essai statistique* sur le royaume de Portugal et d'Algarve [8] [Statistical Essay on the Kingdom of Portugal and Algarve], and also Variétés politico-statistiques de la monarchie portugaise [9] [Various Political-Statistical Observations on the Portuguese Monarchy].



(a) Essai Statistique (b) Variétés Politico-Statistiques (c) Critical remarks

Figure 1: Covers of Balbi's books on the Kingdom of Portugal and Villela da Silva's Critiques

Some statements in Balbi's books probably irritated the portuguese establishment, since in 1828 Villela da Silva [204] published at the Portuguese Royal Press negative critical observations on Balbi's works.

On the other hand, Balbi's work was highly praised by Solano Constâncio (1777-1846), who in 1822 published an analysis of *Variétés politico-statistiques sur la monarchie portugaise* [168] in the *Anais das Ciências, Artes, e Letras* [Annals of Sciences, Arts, and

Letters], Volume XVII, 84–103.<sup>2</sup> It should be noted, incidentally, that Solano Constâncio, editor of the above mentioned *Anais das Ciências, Artes, e Letras, in this critical review* refers to the article *Considerações sobre a Estatística* [Considerations on Statistics] by Cândido Xavier [208], published in 1820, in Volume X, pp. 134–172.

Solano Constâncio was a remarkable expatriate polygraph, who studied Medicine in the United Kingdom between 1791 and 1797, later living in Paris from 1797 to 1799. He returned to Portugal, but in 1807 emigrated again to Paris. From 1822 to 1826 he lived in the United States of America, then returned to Paris, where he died in 1846. Machado de Sousa [98] and Cardoso [25] are interesting sources of information about the wide range of interests of this remarkable intellectual.

Regarding Statistics, in the "Preliminary Discourse" [166] of the Anais das Ciências, das Artes e das Letras, Volume I (1818), pp. 1-37, (pp. 81-99 of Leituras e Ensaios de Economia Política [171] [Readings and Essays in Political Economy]), Solano Constâncio states that

"Statistics, a science equally of our times, will also provide material for some articles and for examining the most notable works or news appearing on this subject." (p. 94)

and on p. 176 of the Anais das Ciências, das Artes e das Letras, in the critical review of An inquiry concerning population, etc..., ou investigação acerca da população e da faculdade de multiplicação da espécie humana; obra destinada a refutar a doutrina do ensaio de M. Malthus sobre esse assunto por W. Godwin, Londres, 1820 [167] [An inquiry concerning population, etc..., or an investigation into the population and the ability to multiply the human species; a work intended to refute the doctrine of Mr. Malthus's essay on this subject by W. Godwin, London, 1820] (Volume XII (1821), 63–104.):

"All the statistical notions that have been collected for a century agree in showing the great uniformity of nature's laws regarding reproduction of the species. The proportion of births and deaths, the number of males and females, the proportion of children per marriage, the number of marriages relative to the total population, and the age proportions, in historically regulated states, are facts that, the more they have been observed, the more conformity has been found between regions, climates, and diverse peoples. Despite the great imperfection of statistical works throughout Europe, those countries where censuses and population maps, birth and death registers have been pursued longer and with more care yield very analogous results among themselves. Among many other important inferences, it uniformly results from all these documents that the human species has a very small tendency to increase regularly and permanently."

<sup>&</sup>lt;sup>2</sup> Pages 212–222 of the 1995 reedition of *Leituras e Ensaios de Economia Política* [171] [Readings and Essays in Political Economy], a selection of works by Solano Constâncio, published by the Bank of Portugal, edited by José Luís Cardoso, accessible at https://www.bportugal.pt/sites/default/files/ocpep-11.pdf. The pages cited ahead are from this edition.

The Armazém de Conhecimentos Úteis, nas Artes e Ofícios, ou Colecção de Tratados, Receitas e Invenções de Utilidade Geral [Repository of Useful Knowledge, in Arts and Crafts, or Collection of Treatises, Recipes, and Inventions of General Utility], J.-P. Aillaud, Paris, 1838, contains "Considerações sobre a estatística" [170] ["Considerations on Statistics"] (1, 51–64), reproduced on pp. 271–277 of Leituras e Ensaios de Economia Política [171] [Readings and Essays in Political Economy]; we quote from p. 276:

*"induction from statistical maps will prove the notable utility of this science, which is now indispensable for governing and enlightening the public."* 

We find it interesting to note that in the 1821 citation [167] the refutation of Malthus' exponential population growth, concluding that "the human species has very little tendency to increase regularly and permanently" foreshadows what Verhulst [198, 199, 200] would later formalize, in 1837, with the logistic model; and that, in the 1838 citation [170], "the notable utility of this science, which is now indispensable for [...] enlightening the public" has a meaning very close to the famous Thomas Carlyle's [27] statement in Chap. II of Chartism: "A judicious man uses statistics, not to get knowledge, but to save himself from having ignorance foisted upon him.".

More interesting still is the use in those excerpts of the words "*inferences*" and "*induc*tion". For Solano Constâncio, Statistics has an inferential component that goes beyond its merely descriptive component. It is perhaps for this reason that he distinguishes "*Es*tatística" [Statistics] from "*Estadística*" [Stadistics], etymologically derived from "Estado" (government), the science of governing or dealing with political affairs, more connected to what is termed official statistics.

It was Francisco Solano Constâncio who, in 1836, in his Novo Diccionario Critico e Etymologico da Lingua Portugueza [169] [New Critical and Etymological Dictionary of the Portuguese Language], defined, for the first time in Portuguese, Statistics as the science "that deals with the enumeration of everything that forms the strength of a nation". He added a list of subjects: population, agriculture, industry, education, public revenues and expenses, military strength, and property distribution. The Novo Diccionario Critico e Etymologico da Lingua Portugueza achieved enduring editorial success, with 11 editions until 1877.

One of the consequences of the 1836 education reform by Passos Manuel was the creation of a Chair of Political Economy at the Faculty of Law (resulting from merging the faculties of *Laws* and of *Canons*) of the University of Coimbra. Adrião Forjaz de Sampaio included in the program of the chair an informal presentation of Descriptive Statistics, from the perspective of preparing official statistics, heavily influenced by the journal of works of the Société Française de Statistique Universelle (Fontes de Sousa, 2005 [51]; Ferrão, 2006 [49]).



Figure 2: Solano Constâncio' s Novo Diccionario Critico e Etymologico da Lingua Portugueza



Thus, the pioneer of Statistics teaching at the Faculty of Law of the University of Coimbra was Adrião Forjaz de Sampaio (1810-1874). He published *Primeiros Elementos da Sciencia da Statistica* [52] [First Elements of the Science of Statistics], in which the influence of Quételet is noted. This manual presents the elements of Descriptive Statistics, giving some prominence to averages and dispersions, with rudimentary mathematical formalism. It is interesting to note that in the title, in later editions Forjaz de Sampaio uses "Stadistics" instead of "Statistics", and that in 1857 José Dias Ferreira (1837-1907), then still a student, published at the University Press an *Ensaio sobre os Primeiros Elementos da Theoria da Estadística do Exmo. Senhor Adrião Pereira Forjaz de Sampaio* [47] [Essay on the First Elements of the Theory of Stadistics by the Honorable Adrião Pereira Forjaz de Sampaio].

In 1887 José Frederico Laranjo [88] published a new program for the Chair of Political Economy and Statistics at the Faculty of Law of the University of Coimbra, "with the approval of Dr. Manuel Nunes Giraldes". Citing Bastien [15] "Note that the Chair of Political Economy, after its creation in 1836, and until 1871, was mostly held by Adrião Forjaz de Sampaio and after that date, and until 1899, by Manuel Nunes Giraldes. Despite the strong relationship of Frederico Laranjo with the Chair of Political Economy, particularly in terms of teaching, program development, publication of teaching manuals, and doctoral thesis examinations [...] he was never awarded the respective chair.". The lessons of J. F. Laranjo in Princípios de Economia Política (1891) [Principles of Political Economy], reprinted in [89], do not contain notes on Statistics. The dichotomy between Estatística/Estadística [Statistics/Stadistics] continued to divide those interested in this new science(s) in the 19th century. On March 26, 1852, Alexandre Herculano<sup>3</sup> (1810-1877) — who was exiled in England and then in France between 1831 and 1832 — proposed to the Lisbon Royal Academy of Sciences that the section of Economic and Administrative Sciences should "draft the necessary instructions and a series of statistical questions, in harmony with the current state of science, which the Academy should bring to the Government's attention, seeking to obtain from it an order for civil, ecclesiastical, and military officials, as well as judicial magistrates and elected authorities of any order and denomination, to respond within the scope of their respective actions to the aforementioned questions, with these responses being transmitted to the Academy."

This interest of Herculano in Statistics was not an isolated episode in his vast array of interests and activities. Two excerpts from *Emigração* [*Emigration*], in *Opúsculos*, Volume IV [79] show that Herculano maintained his interest in Statistics, appreciating its contribution to the objective understanding of social phenomena:

We thus have a set of respectable opinions on the insufficiency of rural wages. These opinions are fully confirmed by statistics, revealing with the irresistible eloquence of numbers the true situation of the laborer, both in terms of his resources and his needs. Incomplete for covering only part of the districts of the kingdom, deficient due to omissions and lack of specification in the elements provided by some municipalities, the statistical tables added to the Parliamentary Inquiry are still numerous enough in their various types to allow general conclusions to be drawn from them.

Alexandre Herculano, Emigration, III in Opúsculos, Volume IV, pp. 150-151.

If the rejection of official means of information seems inconvenient and unfair to me, I find the absolute condemnation of statistics itself, voted to ostracism as a conjectural science, even more peculiar. There is necessarily a misunderstanding here. Statistics aims to collect and methodically organize social facts that can be expressed in numbers. Nothing less conjectural. If, instead of facts, assumptions are collected, that is not statistics, it is fiction. In applied statistics, conjectures and the errors often derived from them are possible; but to infer from this the futility of statistics is the same as denying the scientific validity of arithmetic because sums or multiplications are often mistaken. Generally, what do the laws and measures of a generic nature, whether of a legal, moral, or economic order, depend on? They depend on a correlative statistical study. It is in this study that their raison d'être lies.

<sup>&</sup>lt;sup>3</sup> A remarkable historian, author of an excellent four volumes very comprehensive *História de Portugal* [History of Portugal] and of *História da Origem e Estabelecimento da Inquisição em Portugal*. [History of the Origin and Establishment of the Inquisition in Portugal]. He compiled and edited *Portugaliae Monumenta Historica*, and was also a highly praised romantic novelist author namely of historical novels.

Alexandre Herculano, *Emigration* VII (October 1874) in *Opúsculos*, vol. IV, pp. 212-213.

António de Oliveira Marreca (1805-1889) also went into exile in England and France, returning to Portugal in 1833. He was highly influential in the institutionalization of Political Economy in Portugal, for detailed information cf. Veríssimo Serrão [201] and Almodôvar [3].



In Parecer e Memoria sobre a proposta, que apresentou o Sr. Alexandre Herculano: para que a secção de sciencias económicas e administrativas redigisse um Projecto de Estadística [103] [A Review and Memoir on Mr. Alexandre Herculano proposal, requiring that the section of economic and administrative sciences establishes a Project on Stadistics], Oliveira Marreca returns to the use of the terminology Estadística.

Oliveira Marreca was one of the administrators of the Imprensa Nacional de Lisboa [National Press of Lisbon] from 1834 to 1838. Arguably Oliveira Marreca influenced the publication at the Imprensa Nacional de Lisboa of the translation *Compendio Estadistico ou Resumo dos Elementos de Estadistica* [112] [A Treatise on Statistics, or A Resume of the Elements of Statistics] of the 1847 Éléments de Statistique: comprenant les principes généraux de cette science, et un aperçu historique de ses progrès [111] by A. Moreau de Jonnès (then in charge of the bureau of Statistiques Générales of France). It is worth noting that Jonnès clearly expresses his understanding of the scope and limitations of Statistics:

Statistics is the science of natural, social, and political facts expressed in numbers. Its aim is the deep knowledge of society considered in its nature, elements, economy, situation, and changes. It uses the language of numbers, which is as essential to it as figures are to geometry and symbols are to algebra. It constantly proceeds through numbers, which gives it the precision and certainty characteristic of the exact sciences. Works that bear its name, without having its object or language, do not belong to it, as they fall outside the conditions of its existence. Thus, statistics without numbers, or whose numbers do not express natural, social, and political facts, do not deserve the name they bear. The same can be said of moral and intellectual statistics, because it is a ridiculous claim to want to subject the soul and passions to calculation, and to compute, as if they were defined and comparable units, the movements of the spirit and the phenomena of human intelligence.

For detailed information on Jonnès and his influence, see Paixão Santos [130].

The politician António José de Ávila (1807-1881) — the presumed inspiration of Eça de Queiroz's (1925) posthumous satyrical novel O Conde de Abranhos<sup>4</sup> — also reinforced Alexandre Herculano's proposals in 1854 in a report he prepared for the Minister of Public Works Fontes Pereira de Melo on the works of the Statistical Congress held in Brussels in 1853. He emphasized the importance of establishing uniform bases for national statistical systems, highlighting their importance for the establishment of international comparisons. Cardoso [26] claims that Ávila "supports the need for a general population census every 10 years, with the collection of basic identification and social status information (profession, education, social and health conditions, etc.). Among the subjects that should be the object of statistical standardization, he gives special attention to the following: territory and property registry; emigration and migratory movements; agricultural production; industrial statistics (production, employment, wages, capital, etc.); commercial statistics; budgets of the working classes; census of the indigent (poverty and social assistance); levels of education; criminality and repression." Ávila was again the portuguese official delegate at the Statistical Congress held in Berlin in 1863 [6].

There was also a "folkloric" aspect to the use of Statistics: Sexual taboos were also statistically addressed (Santos Cruz [156]; Azevedo [7]); and perhaps in an attempt to finance their expenses, some travellers published accounts of their peripatetic wanderings with some pretension of statistical objectivity, such as *Portugal a Voo de Pássara* [142]<sup>5</sup>. In the posthumous publication *Londres Maravilhosa* [Wonderful London], in the Notes – Excerpts from the notebooks of M. Teixeira-Gomes<sup>6</sup>, he reveals, with a mixture of surprise and irony, that the novelist and journalist Fialho de Almeida "*is preparing for his excursion to Galiza; he plans to make a complete book with statistics and agronomy about Galiza*!"

The teaching of elementary Probability Calculus, included in the program of Infinitesimal Calculus, was introduced in 1865 at the Escola Politécnica de Lisboa [Lisbon Polytechnic School], formerly Colégio dos Nobres [Nobleman College] and later, in 1911, Faculdade de Ciências [Faculty of Sciences]. Statistics gradually detached itself from its subordination to Political Economy, and in the last quarter of the 19th century, the first research works linked to Mathematics were published in 1866 and 1870 (Daniel Augusto da Silva [160,161]) and later, in 1898, the first doctoral thesis on Mathematical Statistics (Sidónio Paes [129]).

The third posthumous edition (1756) of *The Doctrine of Chances* by Abraham de Moivre [40] contains applications of Probability Theory to actuarial calculations regard-

<sup>&</sup>lt;sup>4</sup>Ávila prohibited a series of liberal conferences (*Conferências do Casino*), and the outstanding novelist Eça de Queiroz used him as model for the ridiculous politician Conde de Abranhos in a satyrical novel, published only 25 years after the death of Eça de Queiroz. As challenging of the political establishment as *Yes, Minister* and *Yes, Prime Minister*.

<sup>&</sup>lt;sup>5</sup> As the novelist Camilo Castelo Branco insultingly translated the title *Le Portugal à Vol d'Oiseau* [*The Flight of a Female Bird in Portugal*,] by Marie Rattazzi, A. Degorce-Cadot, 1880. "Pássara" means literally "female bird", but its slang meaning is "cunning", "smart-ass", "shrewed-ass", or even worse.

<sup>&</sup>lt;sup>6</sup> Aside being a diplomat and President of the Portuguese Republic, Manuel Teixeira-Gomes was a notorious writer, author namely of the novel *Maria Adelaide*, published in 1938, describing the attraction for "nymphets" earlier than the famous *Lolita* of Vladimir Nabokov. Fialho de Almeida was also an important 19th century portuguese novelist, obviously without any serious background in Statistics. In 1905 he published *Cadernos de Viagem. Galiza* [*Travel Notebooks. Galiza*].

ing the computation of annuities. Daniel Augusto da Silva addressed the calculation of annuities in 1866, thus becoming the patron of Portuguese actuaries. It is an interesting investigation but of little statistical interest, the reference to statistics is limited to the excerpt we reproduce.



Os valores da taxa de amortisação, que a estatística e a formula (C) dão para os dois montepios que estudámos — affastam-se ambos, e em sentidos contrarios, das grandezas que esse elemento assumiria n'aquellas duas associações, se ellas se achassem em circumstancias normaes.

The values of the amortization taxes given by Statistics and formula (C) for the two Montepio Mutualist Associations that we analyse fall apart, in opposite directions, from the values they would assume in those associations, in case they would be in normal circumstances.



Daniel Augusto da Silva also published a comparative study of population movements in Portugal [161]. Detailed information can be found in Martins [105], with further complements in Martins [106].

The Geographia e Estatística Geral de Portugal e Colónias [136] [Geography and General Statistics of Portugal and the Colonies] by Gerardo A. Pery, from 1875, is an interesting synthesis that, without methodological innovations, plays a pioneering role in the association of Geography with Statistics. Population is also a major theme in this work, as far as statistics is concerned.





From 1871 to 1899 Manuel Nunes Giraldes was the lecturer in Economics and Statistics at the Faculty of Law in Coimbra; in addition to José Frederico Laranjo's collaboration, M. Emydgio Garcia gave some lectures on the course. Emydgio Garcia published the corresponding notes in 1894 [55], where Statistics was confined to the title of the pamphlet and a set of general comments, which seems to indicate some backlashes in the teaching of Political Economy at the Faculty of Law.

Chapter IV, entitled General Notion of Statistics, pp. 14-17, is merely a set of debatable aphorisms, for instance "Statistics is generally considered the (concrete and descriptive) science that collects, gathers, and records phenomena or facts of observation, expressing them in numbers and geometric signs.

It is not properly a science; it is a necessary aid, an auxiliary, and moreover, an indispensable factor of science."

It is worth mentioning that "mathematical" Statistics had a slow start, mainly because it depends on the language of Probability. The works of John Graunt (1662) [69] and Halley(1693) [78] on mortality are pointed out as the origin of Demography and, indirectly, Actuarial Science, with the referred works of Daniel Augusto da Silva being late descendants.

Other pioneering works in Mathematical Statistics, notably the posthumous publication where Thomas Bayes (1763) [16] used Probability as the language of Statistics for the first time, the great memoir of Laplace's (1774) [87] on inference, and Gauss' research (1809) [56] in which the method of least squares is developed, only came to have an impact on statistical production in Portugal at the end of the 19th century.

The first doctoral thesis at the Faculty of Mathematics of the University of Coimbra in the field of Mathematical Statistics, *Introdução à Teoria dos Erros das Observações* [129] [Introduction to the Theory of Observational Errors], was defended in 1898 by Sidónio Paes. It proposes a rigorous demonstration of the approximation of sums by Gauss's law, and of the use of "lesser squares". It explicitly mentions the works of Bayes, Laplace, Lagrange and Gauss, but without precise referencing.



In fact, in Sidónio Paes thesis, only the books by Poincaré and Bertrand on Probability Calculus, a work by Chauvenet and another by Estienne are identified in footnotes. Works by Encke, Schiaparelli, Stone, Ellis, Tait, Glaisher, Hagen, de Morgan, Ferrero, Bessel and Crofton are mentioned in the text, but without any precise indication.

At the end of the century, Pedro Afonso André de Figueiredo, honoured with the title of Viscount of Wildik, published *Notice Statistique sur le Portugal* [203] for the Universal Exhibition of 1900.

#### Chronology

(English translation of Portuguese titles inside square brackets. Indication of libraries where rare books were found inside angle brackets:  $\langle FLUL \rangle -$  Faculdade de Letras da Universidade de Lisboa;  $\langle FDUL \rangle -$  Faculdade de Direito da Universidade de Lisboa;  $\langle FMUL \rangle -$  Faculdade de Medicina da Universidade de Lisboa;  $\langle IICT \rangle -$  Instituto de Investigação Científica Tropical;  $\langle IST \rangle -$  Instituto Superior Técnico;  $\langle ISA \rangle -$  Instituto Superior de Agronomia;  $\langle FPCEUL \rangle -$  Faculdade de Psicologia e Ciências da Educação da Universidade de Lisboa;  $\langle FCUL \rangle -$  Faculdade de Ciências da Universidade de Lisboa;  $\langle ISCSP \rangle -$  Instituto Superior de Ciências Sociais e Políticas;  $\langle ISEG \rangle -$  Instituto Superior de Economia e Gestão.)

1818 — F. Solano Constâncio's [Preliminary Speech] [166].

1820 — C. Xavier's [Considerations on Statistics] [207].

1821 - F. Solano Constâncio's [An inquiry concerning population, etc..., or an investigation into the population and the ability to multiply the human species; a work intended

to refute the doctrine of Mr. Malthus's essay on this subject by W. Godwin, London, 1820], [167].

1822 — A. Balbi's Essai statistique sur le Royaume de Portugal et d'Algarve, comparé aux autres états de l'Europe, et suivi d'un coup'oeil sur l'État actuel des sciences, des lettres et des beaux-arts parmi les portugais des deux hémishéres [8].

— A. Balbi's Variétés politiques et statistiques de la monarchie portugaise [9].

— F. Solano Constâncio's [Variétés politico-statistiques sur la monarchie portugaise, by A. Balbi] [168].

**1828** — L. D. Villela da Silva's [Critical observations on some considerations in Variétés politiques et statistiques de la monarchie portugaise, published in Paris by Adran Balbi] [204]. < FLUL >

**1836** — F. Solano Constâncio's [New Critical and Etymological Dictionary of the Portuguese Language] [169].

— Passos Manuel's reform of the University, namely creating of a Chair of Political Economy at the Faculty of Law of the University of Coimbra.

1838 — F. Solano Constâncio's [Considerations on Statistics] [170].

1841 — A. P. Forjaz de Sampaio's [First Elements of the Science of Statistics] [52].

- F. I. Santos Cruz's [On the Prostitution at Lisbon] [156].

**1854** — A. Oliveira Marreca's [A Review and Memoir on Mr. Alexandre Herculano proposal, requiring that the section of economic and administrative sciences establishes a Project on Stadistics] [103].  $\langle FLUL \rangle$ 

— A. J. d'Ávila's [Report on the Works of the Congress of Statistics held in Brussels in 1853] [5].

**1856** — A. Moreau de Jonnès' Compendio Estadistico ou Resumo dos Elementos de Estadistica [112], translation of Éléments de Statistique: comprenant les principes généraux de cette science, et un aperçu historique de ses progrès [111].  $\langle ISA \rangle$ 

1857 — J. Dias Ferreira's [Essay on the First Elements of Stadistics Theory] [47]. < FDUL >

1864 — creation of the Direção-Geral de Estatística [General Direction of Statistics].

— A. J. d'Ávila's [Report on the Works of the Congress of Statistics held in Berlin in 1863], [6]. < FDUL >

— F. P. Azevedo's [History of prostitution and sanitary police at Porto, followed by a statistical essay on the two elapsed years, comparative tables, etc.] [7].  $\langle FMUL \rangle$ 

1865 — The teaching of elementary Probability Calculus was included in the program of Infinitesimal Calculus, at the Escola Politécnica de Lisboa [Lisbon Polytechnic School].

1866 — D. A. da Silva's [Average yearly amortizations at the foremost survival Montepio Mutualist Associations in Portugal] [160].

1870 - D. A. da Silva's [Contributions for the comparative study of the population movements in Portugal] [161].  $\langle FDUL \rangle$ 

1875 - G. A. Pery's [Geography and General Statistics of Portugal and the Colonies] [136]. < IICT >

**1887** — J. F. Laranjo's [*Program for the Chair of Political Economy and Statistics at the Faculty of Law of the University of Coimbra*] [88]. < FLUL >

**1894** — M. E. Garcia's [Notes of Some Magistral Lectures in the Course of Political Economy and Statistics] [55].

1898 — Sidónio Paes' [Introduction to the Theory of Observational Errorss] [129].

**1900** — Vicomte de Wildik's [Notice Statistique sur le Portugal]. [203] < FLUL >

# 3 20th Century — From Rodolpho Guimarães (1904) to 1974

Karl Pearson, William Sealy Gosset (*Student*), Ronald Fisher, Jerzy Neyman, and Egon Pearson were major figures in the early developments of Mathematical Statistics, namely in what regards inference about parameters assuming normality, discriminant analysis, and experimental design. These concepts were gradually imported, especially through lecture notes (some published as books) by Varennes e Mendonça at ISA (Instituto Superior de Agronomia), and by Leite Pinto and by Bento Murteira at ISCEF (Instituto Superior de Ciências Económicas e Financeiras). On the other hand, the new methods of Statistics were addressed in a series of publications on "methodological issues" by W. L. Stevens (a disciple of Fisher who worked in the Anthropology Institute of Coimbra between 1942 and 1945), and by his patron E. Tamagnini, and by agronomists influenced by Fisher's school.

Outside the university context, Guimarães [74] used statistical tests to investigate public education issues, and Amaro Guerreiro [72]<sup>7</sup>, a senior civil servant at INE (Instituto Nacional de Estatística) included in his book a description of Pearson distributions (inserts between pp. 246 and 247) and in Chapter 10 (Curve Fitting) in the final section addresses the question of "Fitting to Pearson Type I Curve"; it also contains a chapter (8) on Index Numbers and another (9) on Time Series Analysis.

The teaching of Probability at the Polytechnic School and later at the Faculty of Sciences of Lisbon was crucial to making these developments possible.

#### Probability

Sidónio Paes's 1898 thesis [129], presented at the Faculty of Mathematics of Coimbra University, in a way marked the end of the hegemony of the Faculty of Law at the University of Coimbra in what concerns the university teaching of Statistics in Portugal.

<sup>&</sup>lt;sup>7</sup> An interesting analysis of this book can be found at https://www.deepdyve.com/lp/wiley/ manual-de-estat-stica-vwDem3M8IE?utm\_source=freeShare&utm\_medium=link&utm\_campaign= freeShare.

The research, teaching, and dissemination of Probability Calculus effectively enabled the transition from Descriptive Statistics, sufficient for handling official statistics in the 19th century, to the teaching and research of Inferential Statistics.

Since 1865, a chair of Infinitesimal Calculus including the elementary teaching of Probability Calculus had been taught at the Polytechnic School of Lisbon, and in 1911 a quarterly chair of Probability Calculus was created at the Faculty of Sciences, University of Lisbon, which in 1932 became an annual chair, taught by Victor Hugo de Lemos and Pedro Braumann (cf. Fontes de Sousa, 2005 [51]). The lecture notes [92] from Victor Hugo de Lemos's course was edited in 1945 by J. Cabral Madeira.

The five volumes of Adolpho Loureiro's Os Portos Marítimos de Portugal e ilhas Adjacentes: Atlas [95] [The Sea Ports of Portugal and Adjacent Islands], containing many statistical tables and details, were published from 1904 to 1909.



As far as publications in the area of Probability are concerned, the first noteworthy event was the publication of a booklet intended to the layman, in 1904, volume 223 of the Biblioteca do Povo e das Escolas [Library for the People and the Schools], *Noções sobre Cálculo das Probabilidades, Theoria dos Erros e Méthodo dos Mínimos Quadrados* [75] [Notions of Probability Calculus, Theory of Errors and the Least Squares Method], by Rodolpho Guimarães.

As Rui Santos [150, 154, 155] comments, Rodolpho Guimarães<sup>8</sup>, in the scant 64 pages of this small volume, manages to present an interesting introduction to the topic.

The second notable publication is by José Freire de Sousa Pinto (1855-1911) — the Noções de Cálculo das Probabilidades para o Estabelecimento das Bases da Estatística [173] [Notions of Probability Calculus for Establishing the Bases of Statistics], analysed in Santos [151,153]. José de Sousa Pinto, a professor at the University of Coimbra, discusses the use of Probability Calculus in modeling random phenomena. In this work, the author begins by explaining the process of constructing knowledge in any experimental science, concluding that the advancement of knowledge results from the combination of observing the phenomenon and theoretical reasoning. Thus, any theory arises from the observation of a phenomenon, initially being an approximation. Then, the phenomenon is observed once more. This new observation serves to test the initial theory, which is confirmed or refined with new knowledge from the new observation, or even altered if the initial theory is found to be erroneous. It is in this successive chain of dual effort, observation and reasoning,

<sup>&</sup>lt;sup>8</sup> Rodolfo Guimarães (1866-1918) was a Colonel in the Portuguese army and a professor at the Army School of Lisbon (Military Academy). He also served at the Astronomical Observatory of Tapada in Lisbon. Currently, Guimarães is best known for his research in the History of Mathematics, particularly for publishing a catalog of mathematical works by Portuguese authors during the 19th century and for promoting the work of Pedro Nunes (1502–1578), as well as some contributions in the field of Geometry. For detailed information on Rodolfo Guimarães, see Saraiva [157].

that scientific knowledge advances towards a complete understanding of the phenomenon, where the formalized theory will always occur whenever the phenomenon happens.

It should be noted that the analysis of prerequisites for teaching Statistics was a subject of reflections by Leite Pinto [90], in *Noções de Matemática necessárias ao estudo da Estatística* [Notions of Mathematics Necessary for the Study of Statistics], and by Amaro Guerreiro, in *Em Torno do Ensino da Estatística* [73] [About the Teaching of Statistics], 1952-53.



In 1914, Diogo Pacheco d'Amorim (1888-1976) defended his doctoral dissertation at the University of Coimbra on *Elementos de Cálculo das Probabilidades* [127] [Elements of Probability Calculus], the aim of which was to establish rigorous foundations for the theory of Probability. In fact, this objective failed, and another doctoral thesis on Probability [144], by Manuel Reis (1929), is very critical of Pacheco d'Amorim's thesis.



Later, in a unique work by Luís de Albuquerque [2] on the foundations of the Calculus of Probabilities, those dissertations are not even mentioned. Pacheco d'Amorim's thesis has been exhaustively studied by Rui Santos [152], see also [107].

See, as a simple introduction to the work of Pacheco d'Amorim, Pestana and Velosa [139] and Pestana [138]. There is an annotated English translation of his doctoral thesis by Mendonça *et al.* [107].

In fact, it was only in 1933 that Kolmogorov [86] made an axiomatic construction of Probability, inspired by Maurice Fréchet's ideas on the use of Measure Theory.



Measure Theory was tackled by José Joaquim Dionísio [43] (who, in what regards Probability, published a brief note on Shannon's entropy [44] in *Gazeta de Matemática*), and became the preferred field of work for Pedro Braumann [19, 20].



Pedro Braumann, who was a fellow at Stanford in 1955-56, was also the author of an excellent exposition on the central theme of classical Probability Theory, the arithmetic of probability laws, [Introduction to the Study of Limits of Sums of Independent Casual Variables] [18].

#### Statistics at the University of Coimbra

The teaching of Statistics associated to Political Economy persisted at the Faculty of Law of the University of Coimbra, giving rise to the publication of various texts (Gouveia

et al. [67], Marnoco e Souza<sup>9</sup> [101], Silva and Rocha [162], Rocha and Monteiro [145], Pereira [135]).

On the other hand, at the Anthropology Institute of the University of Coimbra, Eugénio Tamagnini had an innovative perspective on the contribution of Statistics in Biometrics research. In *Revista da Faculdade de Ciências da Universidade de Coimbra*, Tamagnini [181] published an innovative work in Portugal on analysis of variance. Tamagnini also hired W. L. Stevens, a disciple of Ronald Fisher. Xavier da Cunha (1992) [208] comments that "Tamagnini's interest in the statistical treatment of data obtained from various populations and in comparing them led him to hire a foreign professor of statistics, Professor W. L. Stevens, who collaborated with him on blood groups and also organised free courses in statistics for students of the exact and natural sciences. Stevens had been a brilliant disciple and collaborator of Dr. Fisher, having stayed in Coimbra 'making us familiar with statistical methods and their most convenient application', in Tamagnini's words. This initiative was undoubtedly very useful."

The entry Mathematical Statistics at the University of São Paulo: Mr. W. L. Stevens [126] informs that "[W. L. Stevens] then joined the staff of the Statistical Department, Rothamsted Experimental Station, for a few months on urgent war work prior to leaving for Portugal to take up a lectureship in statistics at Coimbra at the request of the British Council and the Foreign Office. In 1944 he returned to Britain and took a post as statistician with Imperial Chemical Industries, Ltd., Billingham. In 1947 he joined the staff of the Admiralty Statistical Department under Mr. H. L. Seal", and it can be conjectured that he had something to do with the British intelligence services during the war.

Fonseca [50] states that "It is in this context of the necessity to improve the statistical treatment of anthropometric, physiological and demographic data, that one understands the arrival of W. L. Stevens, assigned 'as a statistician, to guide the application work of modern statistical methods to Biological Sciences and organise initiation courses for professors and students of the same methods'. The 'Elementary Course of Modern Statistical Methods Applicable to Scientific Investigation', headed by professor W. L. Stevens, functioned between 1942 and 1944, and is linked to a set of published studies in the series 'Questões de Método' ['Methodogical Issues']. [...] As for the years W. L. Stevens spent in Coimbra, the data which allows us to inquire about his teaching activity is frankly scarce. We know that the course he led functioned between 1942 and 1944, but the apparent absence of Stevens' curriculum vitae in the archives of the University of Coimbra, and, moreover, the inexistence of annuals of the same institution for the period between 1943 and 1947 (encompassing, thus, the years during which the course led by W. L. Stevens was lectured), conditioned our investigation. What was the programme of the course? Who attended it?

<sup>&</sup>lt;sup>9</sup> The incomplete copy from FDUL did not allow us to access some parts of the book; it should be noted that it has a section on "the sceptics of Statistics", a subject that is currently widely discussed. In Chapter III (Statistical Laws), sections 59 and 60 are respectively "Law of large numbers" and "Law of small numbers". Poisson, wishing that his result on the convergence of a succession of binomials to Poisson to be known as the law of small numbers, renamed "law of large numbers" Jacques Bernoulli's "golden theorem" on the convergence of arithmetic means towards the expectation. Only the expression "law of large numbers" persisted.

What bibliography was recommended? Why was it extinct after only two years? These are just some of the questions that (at least) for now, will remain without an answer.".

Between 1942 and 1945 Stevens published a remarkable set of works which he categorised as Questions of Method — namely on the  $\chi^2$  significance test, discriminant analysis, statistical distributions, parameters estimation —, disseminating innovations in mathematical statistics by Karl Pearson, 'Student', Ronald Fisher, Egon Pearson, Jerszy Neyman and their schools.

Stevens also published, with Aureliano Quintanilha and Hugo Ribeiro [141] an interesting article on the application of Probability Calculus in the investigation of non sexual reproduction by plasmogamy followed by karyogamy in the higher fungi basidiomycetes, in the *Gazeta de Matemática* (both in the index and in the article the initials W and L are interchanged).

#### Statistics at the Technical University of Lisbon

Several graduates in Mathematical Sciences, Mathematics and Geographical Engineering from FCUL, where they had studied Probability, Errors and Statistics, became professors at institutions that developed the teaching of Mathematical Statistics, namely at Institutes of the Technical University of Lisbon. Special mention should be made of

- Manuel Zaluar Nunes (1907-1967), who started as an assistant at ISCEF, before going to study in Paris as a fellow of the Instituto de Alta Cultura, published in 1933 with Mário Santos a concise exposition of *Elementos de Cálculo das Probabilidades e de Estatística Matemática* [149] [*Elements of Probability Calculus and Mathematical Statistics*]: written in accordance with the program of the 2nd chair of the Instituto Superior de Ciências Económicas e Financeiras. Founder of the *Gazeta de Matemática*, of *Portugaliae Mathematica*, and of the Sociedade Portuguesa de Matemática [Portuguese Mathematical Society], he was an Assistant Professor at FCUL and later a Full Professor at ISA, a position from which he was dismissed in 1947 for political reasons, having to migrate first to France, where he was a research assistant at the Centre National de la Recherche Scientifique, and later to Brazil, where he was hired as a professor at the University of Recife.
- Francisco Paulo Pinto Leite (1902-2000) was lecturer at Sorbonne from 1931 to 1933. One of the founders of the Portuguese Mathematical Society, he was appointed Full Professor at ISCEF in 1940, retiring in 1973. There are various versions of the course notes for the Statistics course he taught at ISCEF, and he was also the first lecturer, in 1952-53, of the chair of Econometrics, with Henri Guitton (University of Dijon) and José de Castañeda (University of Madrid), having as assistant Manuel Jacinto Nunes (further information in Machado and Santos Silva [97]).

It should be noted that 1953 saw the publication of *Trabalhos do Seminário de Econometria Dirigido pelo Prof. H. O. Wold* [206] [*Works presented at the Seminar of Econometry drected by Professor H. O. Wold*] and the translation of Wold's book on dynamic systems [206] by Jacinto Nunes, and that Mira Fernandes (who prefaced

the book by Mário Santos and Zaluar Nunes) consistently supported research in Statistics and Econometrics, see the series of eight volumes of *Estudos de Matemática*, *Estatística e Econometria* [Studies in Mathematics, Statistics and Econometrics] [108] that he edited between 1956 and 1964.

José Sebastião e Silva (1914-1972), a fellow of the Instituto de Alta Cultura in Rome from 1942 to 1946, in 1951 won the public competition for the position of Full Professor at the Instituto Superior de Agronomia, taking charge of the chairs of General Mathematics and of Calculus of Probabilities. In 1954-55, he prepared didactic texts, collected in the course notes Cálculo das Probabilidades [158] [Calculus of Probabilities] published at ISA, with an appendix from 1957-58 on correlation and regression, adjustment by the method of least squares, and Student's t and Fisher-Snedecor's F statistics, cf. Neves [125], pp.222-223. In 2000 Calouste Gulbenkian Foundation republished this work [159], now unfortunately out of print.

Perhaps as a result of the influence of the work developed by Sir Ronald Fisher and his collaborators at Rothamsted, new areas of Statistics were developed at ISA, particularly in the area of Experimental Design (Costa [36], Carvalho [28, 29]) and Linear Models (Pestana [137]).

• Professor Varennes e Mendonça, who graduated in Agronomy Engineering at ISA, taught Probabilities at ISA before the appointment of Professor Sebastião e Silva, publishing a brief course on *Noções de Cálculo das Probabilidades* [194] [Notions of Probability Calculus], for details cf. Neves [125], pp. 224–225. Varennes e Mendonça early on realized the difficulties of translating statistical terms, publishing an interesting article [193] on terminology in 1942.



Murteira and Madureira [122] supplied interesting translations, but it wasn't until much later that the Specialised Committee on Statistical Nomenclature of the Portuguese Statistical Society and the Brazilian Statistical Associationprepared the *Statistics Glossary English-Portuguese* [133], https://spestatistica.pt/glossario, which is periodically updated. See also Ventura [197] et al's [Glossary of Statistical Terms: German, French, English, Portuguese].

# Publishers invest in Statistics — translations and dissemination

The translation, in 1945, of Snedecor's *Statistical Methods Applied to Experiments in Agriculture and Biology* [163] certainly had a major impact on the development of Statistics in Portugal. The popularisation of interest in Probability and Statistics, with an increase in the number of students taking courses in these areas, finally aroused the interest of publishers.



Publicações Europa-América pioneered this move to publish statistical books, with the publication of Gustavo de Castro's books [32, 33] in the collection it Saber [Knowledge] (1959, 1965), an author chosen probably because he had previously published a book on *Inferência Estatística* [31] [Statistical Inference] at LNEC [National Laboratory of Civil Engineering] in 1953.



This was followed by the publication of the translation of Tippett's Statistics [190] in 1955 (reprinted in 1968), and in 1973-75 Europa-América published the two volumes of P. Dagnelie's *Théorie et Méthodes Statistiques: Applications Agronomiques* [37], translated by A. St. Aubyn, and in 1982 the translation of M. L. Lévy's *Comprendre les Statistiques* [96].



Editorial Estúdios COR also ventured out with the publication in 1968 of Gnedenko and Khinchine's book *Elementary Introduction to the Theory of Probability* [58], with preface and translation by A. Simões Neto. Despertar published a translation of one of the highest quality and most successful introductory books, Moroney's *Facts from Figures* [113], in 1969.



## Some highlights

The books by Guimarães [75] and Sousa Pinto [173] were analysed by Rui Santos, and those by Varennes e Mendonça [194], Sebastião e Silva [158] and Pestana [137] by Manuela Neves [125], as already mentioned.

Of the others, we think the publications by Santos and Zaluar Nunes [149], Braumann [18], Reis and Sarmento [143] and Dias Agudo [42] deserve special mention.

The publications by E. Tamagnini [181] and W. L. Stevens [174, 175, 176, 177, 178] in the *Revista da Faculdade de Ciências da Universidade de Coimbra* [Review of the Faculty of Sciences of Coimbra University] also deserve special mention.

• The 1933 book by Santos and Zaluar Nunes [149] has a modernity that contrasts sharply with the 1932 book by Bueno y Martins [23], which has a perspective closer to official statistics.



However, when it comes to descriptive statistics, Bueno y Martins' book is more detailed, for example, in addition to the arithmetic mean, he considers geometric, harmonic and weighted averages.



In what regards graphical representations, Bueno y Martins discusses cartograms, stereograms, presenting various interesting pictograms, and discusses as well as association and contingency, and correlation. It should be noted, however, that some of the more elaborate graphics in Bueno y Martins [23] come from other sources. For example, the extra text between pages 138 and 139 is a reproduction of the cover of the August 1930 issue of the *Boletim da Direção Geral de Estatística* [Bulletin of the General Direction of Statistics], see the *Catálogo Bibliográfico* [Bibliographic Catalogue], p.19, published by INE in 2013 [82]. Bueno y Martins' main statistical reference is the monograph by Yule (the bibliographical sources used are indicated at the end of each section, which is very useful), and it is worth noting that Bueno y Martins indicates several books on Probability in the final bibliography, including A Treatise on Probability by Maynard Keynes.

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#### Part III — Mathematical Statistics.

Chapter I — Fundamental Problems. Chapter II — Schemes. Chapter III — Statistical classification. Chapter IV — Mean values. Chapter V — Correlation. Chapter VI — Covariation.

NOTES. REFERENCES. TABLES. The themes addressed by Santos and Zaluar Nunes are still very elementary, corresponding to a style that has not yet incorporated the ideas of Pearson, Student and Fisher. This book was published before Zaluar Nunes, at the time 26 years old, had his initial period as a *estrangeirado* in Paris. In the bibliographical references, the Probability manuals predominate (Poincaré, 1912; Borel, 1925; Lévy, 1925; Castelnuovo, 1928), and the Statistics references are Yule (1919), Jordan (1927) and Darmois (1928).

As far as statistics is concerned, after some elements of descriptive statistics, they discuss what they call Poisson and Lexis schemes, leading to what they consider to be sub- and super-normal, averages, correlation and covariation. It's an interesting book, outstanding among portuguese books on Statistics at the time of its publication, but it falls far short of what Leite Pinto, Varennes e Mendonça, and Murteira a decade later imported from the new concepts of Mathematical Statistics.

• The publications by W. L. Stevens in *Revista da Faculdade de Ciências da Universidade de Coimbra* [Review of the Faculty of Sciences of the University of Coimbra] are the ones that effectively introduced the new concepts and methods of Mathematical Statistics in Portugal.

"Teoria matemática dalgumas distribuições usadas em estatística" [174] ["Mathematical theory of some distributions used in statistics"], from 1942, discusses random pairs, joint distribution of mean and variance estimators, linear combinations of normal variables, with emphasis on contrasts, and partitioning the  $\chi^2$ . It introduces Student's t variable, and applies Student's t test to regression.

"Estimação estatística" [175] ["Statistical estimation"], published in 1944, is the most interesting from a theoretical point of view, as it introduces the concepts of likelihood, consistency, efficiency, sufficiency, minimum variance and information, always with mathematical details and examples. It also discusses homogeneity and adjustment tests. In the "Summary and bibliography" it refers to the groundbreaking work by Fisher (1922), and claims to supply "for the first time, a demonstration of the methods and formulae" for generalising to the estimation of two or more parameters. He also makes it clear that he "rejects the inverse probability method and the so-called 'Bayes postulate'."

The 1945 publications in vol. XIII are of a more applied nature. In "Aplicação do teste  $X^2$  à Análise da Variância" [176] ["Application of the  $\chi^2$  test to the analysis of variance"] Stevens defines the  $\chi^2$  variable and the corresponding number of degrees of freedom, establishes the additive property and then deals with the decomposition of the  $\chi^2$  and its application to the analysis of variance (a subject already dealt with by Tamagnini in 1937, [181]). The mathematical treatment is detailed.

"Análise Discriminante" [177] ["Discriminant Analysis"] also has an in-depth mathematical treatment, particularly with regard to the joint distribution of estimators. It discusses the binormal distribution, with remarkable graphs, for example for concentration ellipses. Analysis of variance is discussed in detail. There is a final bibliography, referring to work by Ronald Fisher and the author, but at the begining Stevens lists Ronald Fisher, H. Hotelling, Bose, Roy, Mahalanobis and Hsu as mentors of the "new statistical school".

"Novos Métodos para o Estudo da Genética Humana" [178] ["New Methods for the Study of Human Genetics"] deals extensively with the estimation of proportions, discussing *score* tests. Curiously, reflecting trends of the time, and in particular those of his mentor Ronald Fisher and his director in Coimbra, E. Tamagnini, the bibliography, citing Finney, Ronald Fisher, Haldane and Oldrycht, is all from publications in the *Annals of Eugenics*.

• Pedro Braumann was Victor Hugo de Lemos's assistant in *Probabilidades, Erros e Estatística* [Probability, Errors and Statistics], and during his time abroad at Stanford he had the opportunity to refine his knowledge of Measure Theory and the arithmetic of probability laws, in the specific sense of limit laws of sums of random variables, and the inverse problem of decomposing a random variable into independent summands.

His 1958 book, Introdução ao Estudo dos Limites de Somas de Variáveis Casuais Independentes [18] [An Introduction to the Study of Limits of Sums of Independent Casual Variables], deals with stable, self-decomposable and infinitely divisible laws and canonical representations, which were the great "inventions" of Paul Lévy, Kolmogorov and Khinchine, in a rigorous and complete way.

- The Manual de Estatística Médica [143] [Medical Statistics Handbook] by Carlos Santos Reis and Alexandre Sarmento (1960) is most probably the work of self-taught users of Statistics who have studied hard. There are 54 items in the bibliography, some of which (Arkin and Colton, 1956; Bancroft, 1957; Croxton, 1953; Darmois, 1952; Mather, 1959; Snedecor, 1959; Weatherburn, 1947) are well-known statistics texts. It covers an extensive list of topics, see the table of contents reproduced in Appendix 2, from the perspective of the application of Statistics in biomedical areas, and with less emphasis on the concepts of the mathematical theory of Statistics. It is certainly out of date for the actual basic training in Statistics in the area of Life Sciences, but at the time its publication it was top of the range.
- When Dias Agudo returned from his time abroad in the USA, the Biology students in FCUL asked him to teach them a course in Statistics, covering the most necessary results for biologists.

The notes for *Introdução à Estatística* [42] [An Introduction to Statistics], which run to just over 100 pages, discuss probability, random variables, Binomial and Poisson, Normal, the approximation of the central limit theorem; with regard to Statistics itself, there is a brief chapter on Descriptive Statistics, another on estimating the mean value and variance, and finally a chapter on hypothesis testing (mean value, goodness of fit, and comparing two populations), see the table of contents in Appendix 3. Concise and clear, like the masterclasses Professor Dias Agudo taught. Unfortunately, the students who edited the notes (Moreira Campos, Waldemar Nunes) didn't include a bibliography.



#### Foreigners ...

Pedro Teodoro Braumann, born in Munich in 1919, left dangerous Nazi-dominated Germany in 1935 and became a naturalised Portuguese citizen. He graduated (1943) and obtained his doctorate (1951) in Mathematics at FCUL, where he was later a Full Professor, distinguishing himself in teaching and research in Probability Theory. He also taught at the University of Luanda and the University of Aveiro, and ended his career at the FCT of the New University of Lisbon, where he retired in 1989.

As already mentioned, W. L. Stevens was hired by the University of Coimbra between 1942 and 1944, publishing in Portuguese, in *Revista da Faculdade de Ciências da Universidade de Coimbra* [Review of the Faculty of Sciences of the University of Coimbra] a remarkable set of papers on "Questões de Método" [Methodological Issues], which are among the first works on Mathematical Statistics published in Portugal. He also taught free courses in Statistics for students of Exact and Natural Sciences. Unfortunately, the innovative statistical ideas in his work had little influence outside the field of Physical Anthropology.

As it has already been mentioned, in 1953 Henri Guitton (University of Dijon) and José de Castañeda (University of de Madrid) collaborated in the inaugural course in Econometrics at ISCEF, with Leite Pinto and Jacinto Nunes.

#### ... and the estrangeirados circa 1950s

Gustavo de Castro, who graduated in Mathematics from FCUL in 1942, was a scholarship holder in Paris from 1946 to 1948, where he completed the course at the Institut de Statistique, obtaining the highest grade in 1948. Upon returning to Portugal, he worked as a researcher at LNEC and ended his professional career as a professor of Biomathematics at the Faculty of Medicine of Lisbon. In the prestigious collection *Saber* [Knowledge] of Publications Europa-América, he published two popular works on Mathematical Statistics [32] and on Probability [33], as previously mentioned. In 1953, Bento Murteira (1924-2018) passed his doctoral exams on autoregressive processes, supervised by Maurice Kendall, the celebrated co-author of the bible Advanced Theory of Statistics. In London, he also interacted with Tippett and Quenouille, the inventor of the jackknife resampling method. After Leite Pinto, he renewed the teaching of Probability, Statistics, and Econometrics at ISCEF, and his lecture notes widely disseminated Mathematical Statistics in Portugal. Many of his publications were collected in the volume Jubileu – Bento Murteira [119] published by ISEG in 1994.

In the mid-1950s, several Portuguese mathematicians spent a year abroad in the USA. Regarding Statistics, Tiago de Oliveira's internship with one of the leading experts in extreme values, Emil Julius Gumbel, at Columbia University, substantially changed his scientific life, which initially focused on Algebra. His Aggregation exam was based on substantial work on Estatística de Densidades; Resultados Assintóticos [184] [Statistical Distributions; Asymptotic Results], marking his shift towards teaching and research in Probability and Statistics. His 1956 lecture notes on these subjects were the genesis of *Probabilidades e Estatística — Conceitos Fundamentais* [185] [Probability and Statistics — Fundamental Concepts], published in 1967-1969 by Livraria Escolar Editora, later republished in 1990-91 by McGraw-Hill with the title *Probabilidades e Estatística: Conceitos, Métodos e Aplicações* [Probability and Statistics: Concepts, Methods and Applications]. He became one of the most cited Portuguese mathematicians, particularly due to his work in the area of Extreme Value Theory.

For more details, see the *Boletim Informativo da Sociedade Portuguesa de Estatística* [Bulletin of the Portuguese Statistical Society], Special Issue in Honor of Professor José Tiago de Oliveira, December 22, 1998 [1]; also, Gomes [59], The scientific work of J. Tiago de Oliveira, and Gomes [60], The School of Extremes in Portugal — PORTSEA, from English "Portuguese School of Extremes and Applications".

Pedro Braumann's period at Stanford was also significant for the development of teaching and research in Probability in Portugal. Braumann's monograph [18] is a perfect treatise on the classic issues of Probability Theory, and his recurring work on Measure Theory [19, 20] excellently expounds the fundamentals of Probability Theory.

As previously mentioned, F. R, Dias Agudo, whose teaching and research career was more related to Linear Algebra, Mathematical Analysis, and Differential Equations, taught an interesting introductory Statistics course [42] at FCUL upon his return from the USA.

#### Chronology

(In regards to lecture notes, we only indicate one of the years of publication) **1904** — Guimarães' [Notions about Probability Calculus, the Theory of Errors and the Least Squares Method], [75].

1904–1909 — Loureiro's [The Sea Ports of Portugal and Adjacent Islands], [95]. 1909 — Duncker's O Méthodo Estatistico da Variação, translation of [Methode der Variations-Slatistik], [45].

**1910** — Gracias' [A Primer on Statistics], [68] < IICT >

**1911** — A quarterly chair of Probability Calculus was created at the Faculty of Sciences, University of Lisbon.

**1913** — Sousa Pinto [Notions of Probability Calculus for Establishing the Bases of Statistics], [173] < IST >

— Gouveia et al.'s [Lessons of Statistics], [67] < FDUL >

— Marnoco e Souza's [A Course of Statistics] (lecture notes compiled by José Fortes Martinho Simões) [101].  $\langle FDUL \rangle$ , incomplete.

**1914** — Pacheco d'Amorim's [*Elements of Probability Calculus*], [107, 127]

**1919** — Silva and Rocha's [*Statistics; summary of the lectures and questions concerning practical problems*] 1913–1919, [162], Manuscript < FDUL >

1923 — Rocha and Monteiro's [Statistics], 1922–1923, [145], Manuscript  $\langle FDUL \rangle$ 

**1929** — Reis' [*Probability Calculus*], [144].

— Pereira's [Principles of Theoretical and Applied Statistics], [135]. < ISEG/UL >

**1931** — Guimarães' [*Tests: Elementary notions of statistical calculus with applications in scholar measurements*], [74]. < FLUL >

**1932** — An annual chair of Probability Calculus was created at the Faculty of Sciences, University of Lisbon

— Bueno y Martins' [*Principles of General Statistics*] [23].

**1933** — Santos and Zaluar Nunes' [Elements of Probability Calculus and Mathematical Statistics], [149]. < ISEG/UL >

— Costa's [The Technique of Statistical Interpretation of Field Experiments and Mitscherlich Law], [36].  $\langle ISA \rangle$ 

**1935** — Vasconcelos' [Elements of Infinitesimal Calculus and Probability Calculus], [195]  $\langle ISA \rangle$ 

1937 — Tamagnini's [The heterogeneity of variation: analysis of variance] [181]. < FPCEUL >

— Brito's [Statistical Treatment of Geophysical Observations], [21] < FCUL >

1942 — Varennes e Mendonça's [On the Portuguese statistical terminilogy], [193]. < ISA>

— Quintanilha et al.'s [Application of Probability Calculus in solving a problem in Biology], [141].

— Stevens' [Mathematical theory of some distributions used in statistics], [174]. < FPCEUL >

**1944** — Stevens' [Statistical estimation; the theory of estimation of two or more parameters, exemplified with the problem of estimation of the frequencies of genes of blood groups], [175] < FPCEUL >

1945 — Stevens' [Application of the  $X^2$  test to the analysis of variance], [176] (Note: in

the Table of Contents there is an error: vectorial analysis, instead of variance analysis) < FPCEUL >

— Stevens' [*Discriminant Analysis*], [177]. (Note: in the Table of Contents it is attributed to Eugénio Tamagnini; but it is clearly W. L. Stevens' work.) < FPCEUL >

-Stevens' [New Methods for the Study of Human Genetics], [178]. < FPCEUL >

— Carvalho's [Statistics; Modern Methods Appliable to Field Experiments], [28] < ISA >

— Lemos' [Notes on Probability Calculus], [92] < FCUL >

**1945** — Snedecor's Métodos Estatísticos. Aplicados a Experimentação Agrícola e Biológica, translation of [Statistical Methods Applied to Experiments in Agriculture and Biology], [163] < IICT >

— Leite Pinto's [Notions of Mathematics Necessary for the Study of Statistics], [90]. < FCUL >

**1946** — Carvalho's [Statistics and Agricultural Experimentation], [29] < IICT >

1947 - Guerreiro's [Statistics Handbook], [72].

**1949** — Leite Pinto's lecture notes [*Statistical Lessons*], [91].

**1950** — Varennes e Mendonça's [Notions of Probability Calculus], [194] < ISA >

— Baptista's [Discriminant Analysis. A Preliminary study with Amygdalus Communis, L.], [10] < ISA >

**1952** — Guerreiro's [About the Teaching of Statistics], [73] < ISCSP >

**1953** — A chair of Econometry was created at ISCEF.

— Madeira's [Statistical Quality Control], [99].

— Castro's [*Statistical Inference*], [31].

— Soares da Veiga and Ponte's [Introduction to the study of economic statistics], [165].

— Murteira's [Some properties of auto-regressive processes], [115].

— Wold's Sistemas Dinâmicos de Tipo Recursivo: Aspectos Económicos e Estatísticos, translation of [Recursive Dynamical Systems; Economical and Statistical Aspects], [205]

— Wold's Trabalhos do Seminário de Econometria Dirigido pelo Prof. H. O. Wold [Works of the Seminar of Econometry directed by Prof. H. O. Wold] [206].

1954 — Sebastião e Silva's [Probability Calculus], [158]

— Murteira's lecture notes [*Statistics*], [116].

1955 — Tippett's *Estatística*, translation of [Statistics], [190], reprinted in 1968.

**1956** — Tiago de Oliveira's lecture notes [*Probability, Errors and Statistics*], [183].

— Pacheco d'Amorim's lecture notes [Symbolic Calculus and Calculus of Finite Differences. Probability Calculus], [128].

**1956–1964** — Mira Fernandes' [Studies in Mathematics, Statistics and Econometry], 8 volumes, [108].  $\langle IST \rangle$ 

**1957** — Pestana's [Notes on the Estimation and Significance of the Parameters of Normal Linear Models]. [137] < ISA >

— Martinez's [Introduction to an Essay on Economical Statistics], [104]

— Dionísio's [Foundations of Measure Theory], [43].

**1958** — Braumann's [An Introduction to the Study of Limits of Sums of Independent Casual Variables], [18]

— Murteira and Madureira's [Portuguese Statitical Terminilogy], [122] < ISEG >

— Castro's [*Elements of Mathematical Statistics*] (2nd edition in 1983, entitled [*Classical Mathematical Statistics* — the Ideas]), [32]

**1959** — Albuquerque's [Notes on the Foundations of Probability Calculus], [2].

— Dionísio's [The definition of entropy in probability calculus], [44]

— Sá's lecture notes [*Probability, Errors and Statistics*], [180]. < FCUL >

**1960** — Reis and Sarmento's [*Medical Statistics Handbook*], [143] < IICT >

**1961** — Dias Agudo's [An Introduction to Statistics], [42].

**1962** — Lopes' lecture notes [*Probability, Errors and Statistics*], [93].

1963 — Tiago de Oliveira's [Statistical Distributions; Asymptotic Results], [184].

**1965** — Castro's [Introduction to the Theory of Probability], [33].

1967 — Tiago de Oliveira's [Probability and Statistics – Fundamental Concepts], [185].

**1968** — Gnedenko and Khintchine's *Introdução à Teoria das Probabilidades e à Estatística*, translation of [An Elementary Introduction to the Theory of Probability], [58].

**1969** — Braumann's [*Elements of Measure Theory Relevant for Probability Calculus*], [19].

— Moroney's Dos Números aos Factos, translation of [Facts from Figures], [113].

**1971–1973** — Galvão de Mello's [Introduction to Statistical Methods], [54]. (2nd ed. 1993-1997 entitled [Probability and Statistics: Concepts and Fundamental Methods].

## 4 The last 50 years

Fifty years is too short a time to have a historical perspective, and in this section, we will only highlight a few issues that seem to us to be more relevant.

Veiga Simão, who distinguished himself as Rector of the General University Studies of Mozambique between 1962 and 1970, was Minister of Education from 1970 to 1974. He profoundly altered higher education in Portugal, and many of the changes following April 25, 1974, are still a consequence of the deep reform he initiated.

In 1973, the Nova University of Lisbon, the University of Aveiro, the University of Minho, the University Institute of Évora, and the Polytechnic Institute of Covilhã were

established. In 1979, the last two became the University of Évora and the University of Beira Interior. The University of the Azores was created in 1976, the University of Trás-os-Montes and Alto Douro in 1986, and the Open University and the University of Madeira in 1988. Starting in 1979, fifteen public Polytechnic Institutes were established. The 1986 Basic Law of the Education System clarified the objectives of university and polytechnic higher education and the degrees and titles they confer, necessitating the hiring of qualified teaching/research staff.

The political changes of 1974 initially caused a deep crisis in universities, but it is worth remembering that crisis and opportunity form an interesting binary. There was a strong investment in hiring teaching staff and sending young researchers abroad for obtaining their doctorates. By 2000, 38 university teachers from Portugal had obtained doctorates in Statistics abroad, and 4 obtained doctorates in Portuguese universities supervised by foreigners (with an additional 4 doctorates supervised in Portugal, but by assistants who had previously spent a period of postgraduate studies or internships abroad), cf. Amaral Turkman [4].

The Portuguese Statute of the University Teaching Career (Decree-Law 448/79, November 13, ratified by Law 19/80 of July 16) doubled the university staff (expanded with a board of supernumerary staff when necessary, later added to the regular staff). Decree-Law 66/80, of April 9, established the rules for creating departments and autonomous sections in universities, and Decree-Law 263/80, of August 7, legislated the creation of Master's degrees.

Due to the dynamism and prestige of J. Tiago de Oliveira, the Center for Statistics and Applications of the University of Lisbon (CEAUL) was created in 1975, joined by researchers from other universities, and in the Bachelor's degree in Applied Mathematics, courses in Stochastic Processes, Computational Statistics and Simulation were created.

In the early 1980s, FCUL had the critical mass<sup>10</sup> to create the Department of Statistics, Operational Research, and Computing in 1981 (with Computing soon separating), and a Master's course in Statistics and Operational Research. New courses were also created, such as Order Statistics, Computational Statistics, Sampling, Quality Control, Design of Experiments, Reliability, and Non-Parametric Methods.

Starting in the mid-1970s, the University of Coimbra substantially developed the teaching of Statistics with the collaboration of foreign professors, namely Quidel, Delecroix, Moché, Deheuvels, and Teugels. The doctorates of Nazaré Mendes Lopes (1985, supervised by Geffroy), Esmeralda Gonçalves (Lille, 1988), Paulo Oliveira (Lille, 1991), Ana Cristina Rosa (Toulouse, 1994), Emília Nogueira (1993, supervised by Nazaré Mendes Lopes and Delecroix), and Carlos Tenreiro (1995, supervised by Nazaré Mendes Lopes and Gouriéroux) strengthened the area of Statistics in the Department of Mathematics, which housed the Center for Mathematics of the University of Coimbra, homologated by INIC in 1978, with a research line in Probability, Statistics, and Stochastic Processes, directed

<sup>&</sup>lt;sup>10</sup> Doctorates in Sheffield, M. Ivette Gomes (1978), Dinis Pestana (1978), Antónia Amaral Turkman (1980), and Feridun Turkman (1980); and also Cristina Sernadas (London, 1980) and Helena Nicolau and Fernando Nicolau (1981), supervised by Tiago de Oliveira after obtaining the *doctorat de troisième cycle* in Paris.

by Maurice Moché, cf. Tenreiro [182].

In the last quarter of the 20th century, the doctorates in Statistics of teachers from the University of Porto obtained abroad were those of Corália Vicente (Warwick, 1985), Margarida Brito (Paris VI, 1986), Paulo Gomes (Montpellier, 1987), Denisa Mendonça (Exeter, 1987), Carolina Silva (Exeter, 1989), Paula Brito (Paris IX, 1991), Eduarda Silva (Manchester, 1994), Joaquim Pinto da Costa (Rennes I, 1996), and Paulo Teles (Temple, 1999), who boosted the teaching and research of Statistics at ICBAS, Instituto de Ciências Biomédicas Abel Salazar, the Faculty of Sciences, and the Faculty of Economics. Also from Porto, but from the Polytechnic Institute, Fernando Magalhães obtained his doctorate in Sheffield in 1997.

Meanwhile, between 1974 and 2000, the following teachers from the Technical University of Lisbon obtained their doctorates under the supervision of foreign scientists:

- Teachers from ISEG Daniel Muller (at ISEG but supervised by Marie Duflo, 1985), and abroad Lourdes Centeno (Heriot-Watt, Edinburgh, 1985), Nuno Crato (Delaware, 1992), and Alfredo Egídio dos Reis (Heriot-Watt, Edinburgh, 1994).
- Teachers from IST Fernanda Ramalhoto (University College, 1977), João Branco (Newcastle, 1979), Acácio Porta Nova (Austin, 1985), João Amaral (Oxford, 1985), Daniel Paulino (São Paulo, 1989), António Pacheco (Cornell, 1994).
- Teachers from ISA Carlos Agra Coelho (Michigan, 1992), Jorge Cadima (Kent, 1992).

During this period, the doctorates obtained abroad by teachers from the University of Évora were Carlos Braumann (Stony Brook, NY, 1979) and Russell Alpizar-Jara (North Carolina, 1997). From the University of Minho, Pedro Oliveira (Strathclyde, 1992), from the University of Algarve, Paulo Rodrigues (Manchester, 1999). From ISCTE, Manuela Magalhães Hill (Keele, 1987) and Elizabeth Reis (Southampton, 1998).

During this period, some foreigners decided to settle in Portugal: Kamil Feridun Turkman at FCUL; Ludwig Streit, first at the University of Minho and then at the University of Madeira, where he created the CCM – Center for Mathematical Sciences, which is currently a pole of CIMA (Center for Research in Applied Mathematics) at Évora; Alpizar-Jara at the University of Évora; Manuel Scotto, who after obtaining his doctorate in Lisbon with Kamil Feridun Turkman, progressed at the University of Aveiro and later was appointed Full Professor at IST; Giovanni Silva at IST; Laurens de Haan at CEAUL. Any of them had an important impact on Statistics in Portugal. V. V. Yurinsky also settled in the University of Beira Interior but, apart from participating in a congress of the Portuguese Statistical Society, does not seem to have created a useful interrelation with Portuguese statisticians.

Institutions like the Calouste Gulbenkian Foundation, FLAD, and NATO have been important funding and stimulating research in Portugal. With the exchange over the past half-century, including ERASMUS, it would not be unreasonable to say that almost all of us are, to a greater or lesser extent,  $estrangeirados^{11}$ .

The FCT is also an important source of funding for internationalization, through scholarship and project programs, and through research centers. The creation of other research centers in Mathematics, with research lines in Probability, Stochastic Processes, and Statistics (such as the Center for Applied Mathematics (CIMA) at the University of Évora, with a second research line on Stochastic Processes, Statistics, and Operational Research, created in 1994, or the Center for Mathematics (CMAT) at the School of Sciences of the University of Minho (UM), with a branch at the University of Trás-os-Montes and Alto Douro (UTAD), with a research line in Statistics, Applied Probability, and Operational Research) had a huge effect on the internationalization of Portuguese research in Probability and Statistics, financing participation in numerous international events. The "Bulletin of the Portuguese Statistical Society," Spring 2009, published information on several research centers (C. Agra Coelho, M. L. Esquível, and J. T. Mexia on CMA at the New University of Lisbon; M. Scotto on GPE at the University of Aveiro; C. A. Braumann and R. Alpizar-Jara on CIMA at the University of Évora; A. Pacheco on CEMAT at the Technical University of Lisbon; I. Fraga Alves on CEAUL).

Portugal has hosted important international congresses, such as

Recent Advances in Statistics [189], Lisboa.

NATO ASI on Statistical Extremes and Application [186], Vimeiro 1983.

NATO ASI on Stochastic Analysis and Applications on Physics, Funchal 1993.

Applied Stochastic Models and Data Analysis (ASMDA 99), Lisboa 1999.

23rd European Meeting of Statisticians [38], Funchal 2001.

32nd European Mathematical Psychology Group Meeting (EMPG 2001), Lisboa 2001.

VI International Congress on Insurance Mathematics and Econometry, 2002.

IASC-IFCS Joint International Summer School on Classification and Data Mining in Business, Industry and Applied Research – Methodological and Computational Issues (JISS 2003), Lisboa 2003.

III International Conference on Extreme Value Analysis (EVA 2004) [77], Aveiro 2004.

56th Session of the International Statistical Institute [63], Lisboa 2007.

Extremes in Vimeiro Today [53], Vimeiro 2013.

5th International Conference on Risk Assessment (ICRA5) [85], Tomar 2013.

<sup>&</sup>lt;sup>11</sup> Or that only those who don't want to are not *estrangeirados*. Note that there is currently a new breed of foreign-trained statisticians, promising young statisticians who, due to career progression constraints in Portugal, have decided to go abroad, hopefully only for a while. Notable among them are João Manuel Caravana Santos Silva (Essex), Pedro Miranda Afonso (Erasmus Medical Center, Rotterdam), Paulo Canas Rodrigues (Monash University, Melbourne), Miguel de Carvalho (Edinburgh), Lígia Henriques Rodrigues (São Paulo), Cláudia Neves (King's College), Nuno Sepúlveda (Warsaw), and Jorge Sinval (Singapore).

Symbolic Data Analysis Workshop (SDA 2018) [22], Viana do Castelo 2018.

Workshop on New Frontiers in Statistics of Extremes (WNFSE 2020), Lisboa 2020.

17th conference of the International Federation of Classification Societies (IFCS 2022), Porto 2022.

IMS International Conference on Statistics and Data Science (ICSDS) [62], Lisboa 2023.

These conferences brought a large number of foreign statisticians to Portugal, but there is no evidence to determine the effective added value this brought to the progress of Statistics in Portugal. The same can be said of the participation of foreign experts in the monitoring committees of research centres funded by the FCT.

As far as visiting professors are concerned, David Mejzler's stay at FCUL inspired the work of Eugénia Graça Martins and Dinis Pestana [70] on classes of Mejzler, and that of W. Urfer was stimulating for several CEAUL researchers. Vic Barnett's brief stay at FCUL led to the SPRUCE project with Kamil Feridun Turkman, with the publication of



Barnett, V.; Turkman, K. F. (1993). Statistics for the Environment, [12]
Barnett, V.; Turkman, K. F. (1994). Statistics for the Environment 2 — Water Related Issues, [13].
Barnett, V.; Turkman, K. F. (1997). Statistics for the Environment 3 — Pollution Assessment and Control, [14].
Barnett, V.; Stein, A.; Turkman, K. F. (1998).
Statistics for the Environment 4 — Statistical Aspects of Health and the Environment, [11].



The University of Lisbon has awarded doctorates *Honoris Causa* to Vic Barnett, Ross Leadbetter, Laurens de Haan and Bento Murteira, in recognition of their contributions to the development of Statistics research in Portugal. The Universidade Nova de Lisboa awarded a doctorate *Honoris Causa* to C. R. Rao.

#### Scientific societies

Once again, J. Tiago de Oliveira's dynamism led to the creation of the Portuguese Statistical and Operational Research Society (SPEIO), which in 1991 was transformed into the Portuguese Statistical Society (SPE), as the Portuguese Association for the Development of Operational Research had been created in the meantime.

The Portuguese Association for Data Classification and Analysis (CLAD) was also created in 1995.

The SPE regularly organises congresses, usually incorporating a mini-course, see the listing in Appendix 1. Part of the proceedings of these congresses and the mini-courses are available at https://www.spestatistica.pt/pt. CLAD organises conferences (JO-CLAD) with some proceedings available at https://clad.pt/.

The SPE Congresses and JOCLAD are a barometer of statistics research in Portugal. For a co-ordinated overview of research up to 2005, see Murteira (2005) [120].

The SPE elected Sir David Cox and C. R. Rao as foreign honorary members. Honorary members of SPE are also the Portuguese conferred a Life Achievement Award, https://www.spestatistica.pt/en/socios/honorary-members.

Since 2006, Fernando Rosado has been doing invaluable work on what is happening in statistics teaching and research in Portugal, publishing an issue of the Bulletin of the Portuguese Statistical Society [148] in the spring and autumn of each year, choosing a central theme for each issue. They are available at https://www.spestatistica.pt/ publicacoes/categoria/boletim-da-spe.

The themes of all the issues that we owe to Fernando Rosado are:

Autumn 2006	Teaching and Learning Statistics
Spring 2007	The "School of Extremes" in Portugal
Autumn $2007$	Biostatistics
Spring 2008	ALEA - A place of our world
Autumn $2008$	Stochastic Processes
Spring 2009	Research (in) Statistics
Autumn 2009	Econometric Models
Spring 2010	Data Mining
Autumn 2010	Spatial Statistics
Spring 2011	Surveys and Censuses
Autumn 2011	Survival Analysis
Spring $2012$	Statistics in Polytechnic Higher Education
Autumn 2012	Statistical Methods in Medicine
Spring 2013	Non-parametric Statistics
Autumn 2013	The "Bayesian School" in Portugal
Spring 2014	(An) International Year of Statistics
Autumn 2014	Statistics in Primary and Secondary Education
Spring 2015	Statistics in Sport
Autumn 2015	Statistics in Genetics
Spring 2016	Time Series and their Applications
Autumn 2016	The Central Theme of Statistics
Spring 2017	Uncertainty in Engineering
Autumn $2017$	The Central Theme of Statistics — a new look
Spring 2018	Multivariate Statistics — perspective in the 21st century
Autumn 2018	Stochastic Differential Equations and Some Applications
Spring 2019	Integer Value Time Series
Autumn 2019	Statistics in the Health Sciences
Spring 2020	INE - 85 years of statistics serving the country
Autumn 2020	40 years of SPE: Where did we come from? Where are we? Where are we

going?

Spring 2021	Covid Special: Statistics at the service of society
Autumn 2021	Machine Learning and Artificial Intelligence
Spring $2022$	Statistical Leadership
Autumn $2022$	Awards at the Portuguese Statistical Society
Spring 2023	PORTSEA — a Sea of Extremes in Portugal
Autumn $2023$	Statistics (and) Education
Spring $2024$	Rising Stars

The Spring 2008 issues, ALEA – A Place in Our World, the Autumn 2014 issue, Statistics in Basic and Secondary Education, and the Autumn 2023 issue, Statistics (and) Education, address the important issue of pre-university education in Statistics. Sebastião e Silva was a pioneer regarding the teaching of Probability in high schools and had an influence on creating a branch for training Mathematics teachers at FCUL, for which he invited António Simões Neto<sup>12</sup> to the teaching staff, a man of great culture, who influenced several generations of mathematicians (including the older co-author of this essay).

Jaime Carvalho e Silva [30] reminds us of the interest of Bento de Jesus Caraça and Sebastião e Silva in this topic. More recently, António St. Aubyn, Eugénia Graça Martins, Jaime Carvalho e Silva, José Paulo Viana, João Branco, Luísa Canto e Castro Loura, and Manuela Neves, among others, have made significant contributions in this area. However, there have also been unfortunate interventions, and the issue is far from being stabilized (in fact, instability seems to be the hallmark of educational programs ...). Note the investment of INE, ALEA, and Explorística. Incidentally note that the Portuguese Society of Mathematics has a sound investment in students and teachers' mathematical culture with the regular publication of the review *Gazeta de Matemática*: In fact, one of the objectives of the *Gazeta de Matemática* is "to stimulate interest in the study of Mathematics, as well as the exchange of ideas among those who study, teach, research, use, or are simply interested in Mathematics", and consequently, it is a beneficial and stimulating read for innovation in education, including pre-university education. By accessing the Archive https://gazeta.spm.pt/arquivo, and searching for "probability" or "statistics," one can immediately access the articles published in these areas.

Regarding the Spring 2011 issue, Surveys and Censuses, it should be noted that in 1991, the Center for Opinion Studies and Surveys (Universidade Católica) was created.

The Spring 2020 issue, INE – 85 Years of Statistics Serving the Country, addresses the important topic of producing official statistics. For information on the National Statistical System, refer to the interesting book by Ferreira da Cunha [48], and for more information on the INE, other interesting reads are Valério and Bastien [192] and the *Catálogo Bibliográfico* [Bibliographic Catalog] [82].

Regarding the history of statistics in the specific context of official statistics, we recommend Sousa's monograph [172].

Besides its institutional role in producing and disseminating official statistics, INE has supported the progress of Statistics in Portugal and its internationalization. Notably, in

 $<sup>^{12}</sup>$  António Simões Neto was one of the first advocates of the Bayesian perspective. He published little, but see "Coerência subjectivista e teoria ortodoxa das probabilidades" [124].

1996, INE launched the *Revista de Estatística* [Statistical Journal], which was internationalized in 2003 under the name REVSTAT. It can be accessed https://revstat.ine.pt/ index.php/REVSTAT.

The Spring 2021 issue, Special COVID: Statistics at the Service of Society, reminds us of the interesting research in related areas, particularly Epidemiology. Manuel do Carmo Gomes, from the Department of Plant Biology at FCUL, has distinguished himself in this field. The initial phase of Manuel do Carmo Gomes' research was directed by José Manuel Campos Rosado, a Full Professor in the Genetics group at FCUL, who developed the area of Population Dynamics at FCUL.

The Spring 2023 issue focuses on PORTSEA – a Sea of Extremes in Portugal, outlining the research in Extreme Value Theory. The acronym stands for <u>PORT</u>uguese <u>School</u> of <u>Extremes</u> and <u>Applications</u>. The acronym could have been <u>PORT</u>uguese <u>School</u> of <u>EX</u>tremes (PORTSEX), which would have immediately implied its applications.

#### The Booksellers

Statistics is an indispensable tool in a wide range of areas, and many courses now include at least one Statistics class. This has made the publication of books a profitable business, and currently, several publishers are producing what used to be lecture notes (this observation is not intended to be pejorative, it is simply a statement of fact). There is also a focus on publishing exercises with solutions and books illustrating data analysis with the help of statistical software. Notably, Maria Helena Gonçalves and Maria Salomé Cabral [65, 66] have prepared interesting packages for the analysis of longitudinal data, and Camacho and Abreu [24] for Visual Survival Data Analysis.

As we said at the beginning of this section, it is too early to have an idea of what will survive from the multitude of publications in this last half century. The selection criteria for those listed below was that they were useful reference books in various subjects, teaching cycles, or research, but some of them will be forgotten, since they are out of stock and unavailable.

Murteira, B. (1988). [Statistics: Inference and Decision] [117].

Murteira, B.; Muller, D.; Turkman, K. F. (1993). [Analysis of Chronological Sequences], [123].

Tiago de Oliveira, J. (1990-91). [Probability and Statistics — Concepts, Methods and Applications] [187].

Guimarães, R. C. and Sarsfield Cabral, J. A. (1998). [Statistics] [76].

Soares, A. (2000). [Geoestatística para as Ciências da Terra e do Ambiente] [164].

Pestana, D. and Velosa, S. (2002). [Introduction to Probability and Statistics] [140].

Paulino, C. D.; Amaral Turkman, M. A.; Murteira, B. (2003). [*Bayesian Statistics*], [132]. (2nd edition in 2018, with G. Silva as co-author)

Gonçalves, E. and Mendes Lopes, N. (2003). [Statistics: Mathematical Theory and Applications] [64].

Paulino, C. D. and Singer, J. M. (2006). [Analysis of Categorical Data], [134].
Suleman, A. (2009). [Statistical Study of Fuzzy Sets] [179].

Murteira, B. and Antunes, M. (2012). [Probability and Statistics] [121].

It is not very common for Portuguese researchers in this area to publish books in English, but we recommend:

Tiago de Oliveira, J. (1997). Statistical Analysis of Extremes [188].

de Haan, L.; Ferreira, A. (2006). Extreme Value Theory: An Introduction [39].

Turkman, K. F.; Scotto, M.; Bermudez, P. Z. (2014). Non-linear Time Series : Extreme Events and Integer Value Problems [191].

Jacob, D; Neves, C.; Greetham, D. V. (2020). Forecasting and Assessing Risk of Individual Electricity Peaks. [83].

Morais, M. J. C. (2024). Stochastic Processes. Theory, Examples & Exercises [110].

Some publishers continued to publish translations, for example:

Dagnelie, P. (1973–1975). Estatística, Teoria e Métodos, translation of Théorie et Méthodes Statistiques: Applications Agronomiques, [37]

Lévy, M. L. (1982). Introdução à Estatística, translation of Comprendre les Statistiques, [96]

Romano, R. (1989). Enciclopédia Einaudi, vol 15 (Cálculo, Probabilidade), translation of Einaudi Encyclopedia, vol. 15 (Probability, Calculus), [146] (including the chapters Statistical Distribution, Probability, and Decision by Bruno de Finetti, and a chapter by M. Mondadori on Statistical Induction)

Vassereau, A. (1989). Introdução à Estatística, translation of [La Statistique], [196].

D'Hainaut, L. (1991 – 1992). Conceitos e Métodos da Estatística, translation of Concepts et Méthodes de la Statistique, [41].

Hoaglin, D. C.; Mosteller, F.; Tukey, J. W. (1992). Análise Exploratória de Dados. Técnicas Robustas — Um Guia, translation of Understanding Robust and Exploratory Data Analysis, [80]. (It should be noted that he inspired Bento Murteira to rewrite his monograph on Descriptive Statistics, publishing Análise Exploratória de Dados, Estatística Descritiva [Exploratory Data Analysis, Descriptive Statistics], [118].)

Mosteller, F.; Rourke, R. E. K. (1993). *Estatísticas Firmes*, translation of *Sturdy Statistics*, [114].

Ghiglione, R.; Matalon, B. (1993). O Inquérito — Teoria e Prática, translation of Sociological Surveys: Theory and Practice, [57].

60 years late, the famous *How to Lie with Statistics*, by D. Huff was translated<sup>13</sup>. In fact, currently several publishers are publishing lighter entertaining books, for example:

 $<sup>^{13}</sup>$  Recognizing the impact of this book, the magazine *Statistical Science* dedicated in 2005 part of
Clegg, F. (1995). Estatística para Todos: um Manual para Ciências Sociais, translation of Simple Statistics: a Course Book for the Social Sciences, [34].

Marques de Sá, J. (2006). O Acaso — A Vida do Jogo e o Jogo da Vida [Chance: The Life of Games and the Game of Life], [102] (very interesting, we hope that in the reissue the typos will be corrected on p. 33, lines 4–6, where it should be 0.35 instead of 0.45, and on p. 58 in the table the premium case is missing behind 1, "initial choice 1, presenter shows door 3, if you change you lose").

Mlodinow, L. (2009). O Passeio do Bêbado: como o Acaso Rege as nossas Vidas, translation of The Drunkard's Walk: How Randomness Rules our Lives, [109].

Grima, P. (2011). A Certeza Absoluta e Outras Ficções: os Segredos da Estatística, translation of La Certitude Absolue et Autres Illusions : Les Secrets de la Statistique, [71].

Huff, D. (2013). Como Mentir com Estatística, translation of How To Lie With Statistics, [81].

Blauw, S. (2020). O Poder dos Números: Como Estatísticas, Percentagens e Análises nos Enganam e Desenganam, translation of The Number Bias: How Numbers Lead and Mislead Us, [17].

### 5 Final considerations

Although there is no translation of *Statistical Methods for Research Workers* published in Portugal, as far as we were able to ascertain, Fisher's influence was notable on teachers and researchers at the Institute of Anthropology of Coimbra and the Instituto Superior de Agronomia of Lisbon, which introduced in Portugal notions of Analysis of Variance and Experiment Planning, properties of estimators and significance tests.

The Faculty of Law of the University of Coimbra, where Political Economy included elementary notions of Statistics, was more involved with Descriptive Statistics and official statistics. Mathematical Statistics only appeared timidly with the notes by M. Santos and Zaluar Nunes, later developed by Leite Pinto and especially by Bento Murteira.

The Faculties of Science and the Higher Institutes of the Technical University of Lisbon were pivotal in the teaching of Probability, Errors, and Statistics, although, with some justification, Fernandes Costa [46] lamented "the name given to the new discipline, stigmatizing it from birth". Pedro Braumann, after initial training as an assistant in this subject at FCUL, became the researcher with the most advanced works in Probability Theory

<sup>20 (3)</sup> to a "SPECIAL SECTION: HOW TO LIE WITH STATISTICS TURNS FIFTY", with articles by J. M. Steele "Darrell Huff and Fifty Years of How to Lie with Statistics", J. Best "Lies, Calculations and Constructions: Beyond How to Lie with Statistics", M. Monmonier "Lying with Maps", W. Krämer and G. Gigerenzer "How to Confuse with Statistics or: The Use and Misuse of Conditional Probabilities", R. D. De Veaux and D. J. Hand "How to Lie with Bad Data", C. Murray "How to Accuse the Other Guy of Lying with Statistics", S. C. Morton "Ephedra", S. E. Fienberg and P. C. Stern "In Search of the Magic Lasso: The Truth About the Polygraph", https://projecteuclid.org/journals/statistical-science/volume-20/issue-3.

following his internship at Stanford. More applied areas, such as Quality Control and Reliability or Queueing Theory, have significant prominence, particularly in Engineering Institutes. Biostatistics, Survival Analysis, Clinical Trials, and Meta-Analysis are increasingly important in Life Sciences applications. In addition to the Faculties of Science and the Institute of Agronomy, noteworthy groups in this field include those at the University of Minho and the Abel Salazar Institute of Biomedical Sciences, the Faculties of Medicine, and the National School of Public Health.

Spatial Statistics has also seen significant development, particularly at FCUL and IST of the University of Lisbon, at FCT and IMS of the New University of Lisbon, at the University of Évora, and at the University of Minho. The Open University has invested in the area of Experimental Design, and the Catholic University, as previously mentioned, in Sampling. Both the University of Porto and the University of Coimbra have internationally renowned groups in the aforementioned areas and beyond, such as Stochastic Processes, Time Series, and Densities Estimation, spread across their various institutions. Environment Statistics is also a field in development in various universities, and Population Dynamics and Epidemics, for instance in the University of Lisbon, or Fisheries Stocks (Évora, Algarve), and several areas of Stochastic Analysis and of Stochastic Differential Equations (Évora, Lisboa).

The works of M. Ivette Gomes on the History of Statistics highlights that Extreme Value Theory refers important researchers in this area at Universities and Polytechnic Institutes in Portugal [61]. In this brief overview, undoubtedly filled with unjust omissions, we only mention the institutions where Statistics groups have reached a critical mass to stand out, demonstrating the need for investment to reward the work already done at the Universities of Aveiro, Algarve, Azores, Madeira, Beira Interior, and UTAD, as well as at the Polytechnic Institutes, allowing them to gain visibility. In any case, comparing the current situation, both in terms of the number of faculty/researchers and the educational offerings, with what was happening fifty years ago, it is evident that we must thank our mentors, who paved the initial way, and commend the visionary spirit of Veiga Simão, who profoundly transformed higher education in Portugal.

After the brief foray by Daniel Augusto da Silva into Actuarial Calculus, mention should be made of Arthur Malheiros' book [100], available at the Insurance and Pension Funds Supervisory Authority. Regarding the teaching and research in Actuarial Calculus, refer to Lourdes Centeno [94] for the history of its teaching in Portuguese universities.

It is noteworthy that Statistics continues to attract scientists from diverse backgrounds (for example, Carlos Daniel Paulino started as a chemical engineer, Tiago Marques and Manuel Carmo Gomes are biologists, and J. M. Campos Rosado was initially a veterinarian), and it develops in various institutions — and we should not forget the Bank of Portugal — which definitively contributes to its versatility.

Varennes e Mendonça was not the only distinguished professor from other fields who, by necessity, accepted the chair of Probability. To our knowledge, and certainly missing others, Pedro Lago at the Faculty of Sciences of the University of Porto and Manuel Neto Murta in Coimbra also chaired Probability courses. The Mathematics Department of the University of Coimbra, honoring the good tradition of settling its debts, organized the 1989 Probability and Statistics Days in honor of Professor Manuel Neto Murta, publishing *Estudos de Probabilidades e Estatística* (Probability and Statistics Studies) with the works presented by C. Gourieroux and I. Peaucelle, R. Moché, M. I. Gomes, E. Gonçalves, M. Delecroix, M. N. Mendes Lopes, and J. Tiago de Oliveira.

Sampling and Experimental Design are still not fundamental courses for good training in scientific research methodology at many higher education institutions. The 2001 book by Vicente *et al.* [202] is an obvious recommendation for anyone wanting to learn the most common sampling techniques, as is the SPE mini-course by Paulo Gomes, *Tópicos de Sondagens* [Survey Topics], available at https://spestatistica.pt/publicacoes/ publicacao/topicos-de-sondagens. The famous book by Kinsey *et al.* [84] (Portuguese translation published by Meridiano, but to our knowledge out of print) has a remarkable chapter on Sampling, with many judicious observations on, for example, the art of interviewing. Incidentally, we recall that Kinsey's book provoked great controversy, leading the American Association of Statistics to request renowned statisticians to publish a report evaluating the quality of the data collection and analysis. The result, Cochran *et al.* [35] is a notable book that we recommend for developing the critical thinking of all Statistics users.

With the growth of human resources, universities have created departments, Master's and Doctoral programs in Probability and Statistics and related areas. Notably, the Institute of Statistics and Information Management (ISEGI) was established in 1989 at the New University of Lisbon, with strong ties to INE, later renamed NOVA IMS – Information Management School, and the degree in Applied Mathematics to Economics and Management (MAEG) at ISEG/UTL. The Francisco Manuel dos Santos Foundation, established in 2009, has also played a significant role in disseminating statistical data about Portugal through PORDATA (https://www.pordata.pt/portugal).

The preparation of this work led us to realize how inaccessible works of our scientific heritage are. It would certainly be reasonable to digitize these "reserved" works and eventually make them accessible on the SPE website, or even suggest to the FCT or the Academy of Sciences to support a project to create a digital library that would allow access to those interested in the History of Science.

Additionally, some books published in the last quarter-century are unavailable because the Calouste Gulbenkian Foundation holds the copyright and does not reissue them (for instance the works of Sebastião e Silva [159] and the book by Pestana and Velosa [140], and because Livraria Escolar Editora has disappeared (namely the books by Esmeralda Gonçalves and Nazaré Mendes Lopes [64], and by Bento Murteira and Marília Antunes [121]). Surely an institution like SPE or the centers of those authors could reach an agreement for the rights to reprint, at least in PDF.



"Bento Murteira's more daring work. *Estatística: Inferência e Decisão* [Statistics: Inference and Decision], published in 1988, due to the deep critical reflection on the statistical methodologies it constitutes, was in our midst a milestone that promoted a critical awareness of statistical theory and practice that, even today, must be a regular and reflective read for those who make Statistics their professional occupation." — Daniel Paulino (2005) [131].

Once again, a call for republication: surely accredited institutions could insist with Imprensa Nacional – Casa da Moeda to reprint, or possibly transfer the rights to an institution that would do so, since the 2024 centenary reissue is limited to 100 books intended to be offered to libraries.

Finally, to lighten so much scientific bibliography: references to Probability and Statistics in literature are an interesting indicator of the social recognition of these disciplines. Already in the 19th century, Sherlock Holmes praises Probability:

"We balance probabilities and choose the most likely. It is the scientific use of the imagination." Arthur Conan Doyle, The Hound of the Baskervilles.

Meanwhile, Mark Twain (who also contributed with a "data analysis" of sorts, "quitting smoking is easy; I've done it over a hundred times") attributes — it is a lie — the aphorism "There are three kinds of lies: lies, damned lies, and statistics" to Disraeli.

Isaac Asimov's *Foundation*, reputedly the most famous science fiction series, is driven by "psychohistory", a new branch of Probability, and in particular, the second volume, *Foundation and Empire*, extensively discusses risk and the management of low probabilities. It ensures a pleasurable read.

An Instance of the Fingerpost by Ian Pears, translated into Portuguese as "O Círculo da Cruz" (Livros do Brasil), is a fascinating detective novel with a very interesting description of the invention of the experimental method, foreshadowing Fisher's experimental design.

A Conspiracy of Paper by David Liss features a hero, Benjamin Weaver, a Jew of Portuguese descent who becomes embroiled in the financial fraud of the South Sea Company. Being an amateur probabilist, the book contains a set of interesting information about intellectuals who dealt with the early developments of Probability in the United Kingdom, less famous than the exiled Frenchman Abraham de Moivre. A fascinating read.

# Appendix 1

Short Courses, Congresses of the Portuguese Statistical Society, downloadable at https://www.spestatistica.pt/en/publications/category/short-course

Gomes, P. (1998) Tópicos de Sondagens [Survey Topics]

Gomes, M. I.; Figueiredo, F.; Barão, M. I. (1999) 2ªed. 2010, Controlo Estatístico da Qualidade [Statistical Quality Control]

Amaral Turkman, M. A.; Silva, G. (2000) *Modelos Lineares Generalizados — da Teoria* à *Prática* [Generalized Linear Models — from Theory to Practice]

Brilhante, M. F.; Pestana, D.; Rocha, J.; Rocha, M. L.; Velosa, S. (2001) 2<sup>a</sup> ed. 2011, Inferência Estatística sobre a Localização Usando a Escala [Statistical Inference about Location Using the Scale]

Muller, D. (2002) Modelos Heterocedásticos. Aplicações com o software Eviews [Heteroscedastic Models. Applications with Eviews software]

Gonçalves, E.; Mendes Lopes, N. (2003) 2<sup>a</sup>ed. 2008 *Séries Temporais: Modelações Lineares e não Lineares* [Time Series: Linear and Non-Linear Modeling]

Branco, J. A. (2004) Uma Introdução à Análise de Clusters [An Introduction to Cluster Analysis]

Braumann, C. (2005) Introdução às Equações Diferenciais Estocásticas e Aplicações [Introduction to Stochastic Differential Equations and Applications]

Rosado, F. (2006) Outliers em Dados Estatísticos [Outliers in Statistical Data]

Pires, A. M.; Branco, J. A. (2007) Introdução aos Métodos Estatísticos Robustos [Introduction to Robust Statistical Methods]

Carvalho, M. L.; Natário, I. C. (2008) Análise de Dados Espaciais [Spatial Data Analysis]

Rocha, C.; Papoila, A. L. (2009) Análise de Sobrevivência [Survival Analysis]

Tenreiro, C. (2010) Uma Introdução à Estimação Não-Paramétrica da Densidade [An Introduction to Non-Parametric Density Estimation]

Cabral, M. S.; Gonçalves, M. H. (2011) Análise de Dados Longitudinais [Longitudinal Data Analysis]

Salgueiro, M. F. (2012) *Modelos com Equações Estruturais* [Models with Structural Equations]

M. Ivette Gomes, M. I.; Fraga Alves, M. I., Neves, C. (2013) Análise de Valores Extremos: uma Introdução [Extreme Value Analysis: an Introduction]

Amaral Turkman, M. A.; Paulino, C. D. (2015) Estatística Bayesiana Computacional — uma Introdução [Computational Bayesian Statistics — an Introduction]

Brilhante, M. F. (2017) Uma Introdução à Meta-Análise [An Introduction to Meta-Analysis]

Amado, C.; Nunes, C.; Sardinha, A. (2019) Análise Estatística de Dados Financeiros [Statistical Analysis of Finance Data]

Afonso, P. M. (2023). *Modelação Conjunta de Dados Longitudinais e de Sobrevivência* [Joint Modeling of Longitudinal and Survival Data] Before the SPE Congresses, CEAUL organized conferences, whose proceedings were published:

Actas, I Colóquio de Estatística e Investigação Operacional, Lisboa, Centro de Estatística e Aplicações, 1977.

Actas, II Colóquio Estatística e Investigação Operacional, Fundão/Covilhã, CEAUL/ SPEIO, 1981.

Actas, III Colóquio Estatística e Investigação Operacional, CEAUL/SPEIO, Tróia, 1985.

Proceedings of the SPE Congresses:

A Estatística e o Futuro e o Futuro da Estatística [Statistics and the Future and the Future of Statistics], Actas do I Congresso da SPE, 1993, Vimeiro; ed. Dinis Pestana, Antónia Turkman, João Branco, Luísa Canto e Castro e Ana Pires.

Actas do II Congresso da SPE, 1994, Luso; ed. M. Nazaré Mendes Lopes, M. Esmeralda Gonçalves, M. Emília Nogueira, Ana Cristina Rosa e Helena Ferreira.

Bom Senso e Sensibilidade: Traves Mestras da Estatística [Common Sense and Sensitivity: Main Supports of Statistics], Actas do III Congresso da SPE, 1995, Guimarães; ed. João Branco, Paulo Gomes e Jorge Prata.

A Estatística a Decifrar o Mundo [Statistics Deciphering the World], Actas do IV Congresso da SPE, 1996, Funchal; ed. Rita Vasconcelos, Isabel Fraga Alves, Luisa Canto e Castro e Dinis Pestana.

*Estatística – a Diversidade na Unidade* [Statistics – Diversity in Unity], Actas do V Congresso Anual da Sociedade Portuguesa de Estatística, 1997, Curia; ed. Manuela Souto de Miranda e Isabel Pereira.

Afirmar a Estatística: um Desafio para o Século XXI [Asserting Statistics: a Challenge for the 21st Century], Atas do VI Congresso da SPE, 1999, Tomar; ed. Carlos Daniel Paulino, António Pacheco, Ana Pires e Ferreira da Cunha.

*Um Olhar sobre a Estatística* [A Look upon Statistics], Actas do VII Congresso da SPE, 2001, Ofir; ed. Pedro Oliveira e Emília Athayde.

A Estatística em Movimento [Statistics in Motion], Actas do VIII Congresso da SPE, 2001, Peniche; ed. Maria Manuela Neves, Jorge Cadima, Maria João Martins e Fernando Rosado.

Novos Rumos em Estatística [New Directions in Statistics], Actas do IX Congresso da SPE, 2002, Ponta Delgada; ed. Lucília Carvalho, Fátima Brilhante e Fernando Rosado.

*Literacia e Estatística* [Literacy and Statistics], Actas do X Congresso da SPE, 2003, Porto; ed. Paula Brito, Adelaide Figueiredo, Fernanda Sousa, Paulo Teles e Fernando Rosado.

*Estatística com Acaso e Necessidade* [Statistics with Chance and Necessity], Actas do XI Congresso da SPE, 2004, Faro; ed. Paulo M. M. Rodrigues, Efigénio da Luz Rebelo e

Fernando Rosado.

*Estatística Jubilar* [Jubilar Statistics], Actas do XII Congresso Anual da SPE, 2005, Évora; ed. Carlos Braumann, Paulo Infante, Manuela M. Oliveira, Russell Alpízar-Jara e Fernando Rosado.

A Ciência Estatística [Statistical Science], Actas do XIII Congresso da SPE, 2006, Ericeira; ed. Luísa Canto e Castro, Eugénia Graça Martins, Cristina Rocha, M. Fernanda Oliveira, Margarida Mendes Leal e Fernando Rosado.

*Estatística, Ciência Interdisciplinar* [Statistics, an Interdisciplinary Science], Actas do XIV Congresso da SPE, 2007, Covilhã; ed. Maria Eugénia Ferrão, Célia Nunes e Carlos Braumann.

*Estatística: da Teoria à Prática* [Statistics: from Theory to Practice], Actas do XV Congresso da SPE, 2008, Lisboa; ed. Manuela Magalhães Hill, Manuel Alberto Ferreira, José G. Dias, Maria de Fátima Salgueiro, Helena Carvalho, Paula Vicente e Carlos Braumann.

*Estatística: Arte de Explicar o Acaso* [Statistics: the Art of Explaining Chance], Actas do XVI Congresso Anual da SPE, 2009, Vila Real; ed. Irene Oliveira, Elisabete Correia, Fátima Ferreira, Sandra Dias e Carlos Braumann.

Advances in Regression, Survival Analysis, Extreme Values, Markov Processes and Other Statistical Applications, Selected Papers of the Statistical Societies, Springer, XVII Congresso Anual da SPE, 2013, Sesimbra; Ed. João Lita da Silva, Frederico Caeiro, Isabel Natário, Carlos A. Braumann, Manuel L. Esquível and João Tiago Mexia.

Recent Developments in Modeling and Applications in Statistics, Selected Papers of the Statistical Societies, Springer, XVIII Congresso Anual da SPE, 2013, São Pedro do Sul; ed, Paulo Eduardo Oliveira, Maria da Graça Temido, Carla Henriques e Maurizio Vichi.

New Advances in Statistical Modeling and Applications, Selected Papers of the Statistical Societies, Springer, XIX Congresso Anual da SPE, 2014, Nazaré; ed. António Pacheco, Rui Santos, Maria do Rosário Oliveira e Carlos Daniel Paulino.

*Estatística: Novos Desenvolvimentos e Inspirações* [Statistics: New Developments and Inspirations], Actas do XX Congresso Anual da SPE, 2013, Porto; ed. Manuela Maia, Pedro Campos e Pedro Duarte Silva.

*Estatística: a ciência da incerteza* [Statistics: the science of uncertainty], Atas do XXI Congresso da SPE, 2014, Aveiro; ed. Isabel Pereira, Adelaide Freitas, Manuel Scotto, Maria Eduarda Silva e Carlos Daniel Paulino. https://www.spestatistica.pt/publicacoes/ publicacao/estatIstica-ciencia-incerteza

*Estatística: progressos e aplicações* [Statistics: Progresses and Applications], Atas do XXII Congresso da SPE, 2015, Olhão; ed. Clara Cordeiro, Conceição Ribeiro, Carlos Sousa, Maria Helena Gonçalves, Nelson Antunes e Maria Eduarda Silva. https://www.spestatistica.pt/publicacoes/publicacao/estatIstica-progressos-aplicacoes

Atas do XXIII Congresso da SPE [Proceedings of the XXIII SPE Congress], 2017, Lis-

boa; ed. Maria de Fátima Salgueiro, Paula Vicente, Teresa Calapez, Catarina Marques e Maria Eduarda Silva. https://www.spestatistica.pt/publicacoes/publicacao/ atas-do-xxiii-congresso-da-spe

*Estatística: Desafios Transversais às Ciências com Dados* [Transversal challenges to Sciences with Data], Atas do XXIV Congresso da Sociedade Portuguesa de Estatística, 2021, Amarante ed. Paula Milheiro, António Pacheco, Bruno de Sousa, Isabel Fraga Alves, Isabel Pereira, Maria João Polidoro e Sandra Ramos. https://www.spestatistica.pt/ publicacoes/publicacao/estatistica-desafios-transversais-ciencias-com-dados

Recent Developments in Statistics and Data Science, Atas do XXV Congresso da Sociedade Portuguesa de Estatística, 2021, Évora (ed. Regina Bispo, Lígia Henriques-Rodrigues, Russell Alpizar-Jara e Miguel de Carvalho). https://www.spestatistica. pt/publicacoes/publicacao/recent-developments-statistics-and-data-science

New Frontiers in Statistics and Data Science, Springer Proceedings in Mathematics & Statistics, Springer, XXVI Congresso da SPE, 2024, Guimarães (ed. Lígia Henriques-Rodrigues, Raquel Menezes, Luís Meira Machado, Miguel de Carvalho e Susana Faria).

# Appendix 2

Table of Contents of the Manual de Estatística Médica [143] [Manual of Medical Statistics] by Carlos Santos Reis and Alexandre Sarmento.

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## Appendix 3

Table of Contents of Introdução à Estatística [42] [Introduction to Statistics], notes for the course on General Mathematics for the degrees in Biological Sciences, Geological Sciences, and Adjunct Professors, based on the lectures of Professor Dias Agudo, compiled by Moreira Campos / Waldemar Nunes, Lisbon 1961/62

Chapter I – Fundamental Concepts of Probability Theory

- 1.1 Introduction
- 1.2 A fundamental principle
- 1.3 "Arrangements and Combinations"
- 1.4 Event space and probability space
- 1.5 Probabilities of the sum and product of events. Conditional probabilities. Incompatible and independent events
- 1.6 Problem with repeated trials.
- Binomial formula

Chapter II – Random Variables and Distribution Functions

- 2.1 Casual variables
- 2.2 Distribution function
- 2.3 Classification of distributions
- 2.4 Functions of casual variables
- 2.5 Percentiles of a distribution

Chapter III – Discrete Distributions

- 3.1 Probability function; elementary probabilities
- 3.2 Mean value
- 3.3-Moments
- 3.4 Important examples of discrete distributions Poisson distribution Binomial Distribution

Chapter IV – Continuous Distributions

- 4.1 Probability density
- 4.2 Mean value. Moments. Median
- 4.3 Important example of continuous distribution: Normal distribution
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  - 5.1 Mean value and variance of the sum
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#### Chapter VI - Sampling

Data Presentation and Description

- 6.1 Introduction
- 6.2 Frequency distribution; histograms; data classification
- 6.3 Location and dispersion measurements

- Chapter VII Parameter Estimation
  - 7.1 Introduction
  - 7.2 Mean value estimation
  - 7.3 Variance estimation

Chapter VIII – Hypothesis Tests

- 8.1 Introduction
- 8.2 Hypothesis tests on the mean value
- 8.3 Hypothesis tests on the type of distribution
- 8.4 Comparison of two populations

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