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On generalized derivations of partially ordered sets

Ahmed Y. Abdelwanis, Abdelkarim Boua

Abstract. Let P be a poset and d be a derivation on P. In this research, the notion of generalized d-derivation on partially ordered sets is presented and studied. Several characterization theorems on generalized d-derivations are introduced. The properties of the fixed points based on the generalized d-derivations are examined. The properties of ideals and operations related with generalized d-derivations are studied.

1 Introduction

The notion of derivation, extracted from the analytic theory, helps to study in detail the structure and property in an algebraic systems. It is possible to be applied to every algebraic structure endowed with two binary operations. Initially, work on derivation concentrate on rings, near-rings and also on prime rings.

Derivation is an important area of research in the theory of algebraic structure in pure mathematics. The theory of derivations of algebraic structures appeared from the process of developing of Galois theory and the theory of invariants. Several researches have been work on derivations on different algebras and ordered sets. The notion of derivation is related with the notion of generalized derivation few years ago. There are many articles that talk about generalized derivations on rings and rings close (for example see the following references [5] and [6]). Also many papers have talked about derivations, generalized derivations over lattices as [1], [2], [8], [10], [11], [12].

In [13] the notion of derivations on partially ordered sets is introduced and studied. Several characterization theorems on derivations are presented.

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Affiliation:
Ahmed Y. Abdelwanis – Department of Mathematics, Faculty of Science, Cairo University, Giza 12613, Egypt
E-mail: ayunis@sci.cu.edu.eg
Abdelkarim Boua – Sidi Mohammed Ben Abdellah University, Polydisciplinary Faculty, LSI, Taza; Morocco
E-mail: abdelkarimboua@yahoo.fr

In this work, we present the notion of generalized d-derivation and study the related properties of generalized d-derivation on partially ordered sets. Also, we give a few notations for the rest of the part.

In the following, $(P, \leq)$ denotes a partially ordered set (poset). We additionally utilize the shorthand P to indicate a poset. For $y \in P$, we write

$$\downarrow y = \{p \in P : p \leq y\}$$

and

$$\uparrow y = \{p \in P : y \leq p\}.$$

For $B \subseteq P$, we denote

$$l(B) = \{p \in P : p \leq b \text{ for all } b \in B\}$$

the lower cone of B and

$$u(B) = \{p \in P : b \leq p \text{ for all } b \in B\}$$

the upper cone of A dually. It is quickly clear that both are antitone and their compositions $l(u())$ and $u(l())$ are monotone. Also from [3] we have $l(u(l())) = l()$, $u(l(u())) = u()$. If $B = \{b_1, b_2, ..., b_n\}$ is a finite subset, then we write $l(B) = l(b_1, b_2, ..., b_n)$ and $u(B) = u(b_1, b_2, ..., b_n)$ simply. Also, for $A \subseteq P$ and $B \subseteq P$, we will denote $l(A, B)$ for $l(A \cup B)$ and $u(A, B)$ for $u(A \cup B)$. For $A \subseteq P$, we write

$$\downarrow A = \{p \in P : p \leq a \text{ for some } a \in A\}.$$

The paper is sorted out as takes after. In Section 3, we present the notion of generalized d-derivations of partially ordered sets and concentrate their essential properties. In Section 4, we examine the fixed sets in light of the generalized d-derivation. In Section 5, we examine the properties of ideals and the operations related with the generalized d-derivation.

2 Some preliminaries

In this section we recall some definitions which are useful in the next sections.

Definition 1 ([9]). Let P be a poset and $\alpha\colon P \to P$ a function. α is called meet-translation on P, if for all $x, y \in P$, $\alpha(l(x, y)) = l(\alpha(x), y)$.

Definition 2 ([7]). Let P be a poset and $\alpha\colon P \to P$ a function. α is called lower homomorphism on P, if for all $x, y \in P$, $u(\alpha(l(x, y))) = u(l(\alpha(x), \alpha(y)))$.

Definition 3 ([4]). Let P be a poset, $B \subseteq P$ and $a \in P$, a is an upper bound if $b \leq a$ for all $b \in B$. A subset A of P is directed if it is nonempty and every finite subset of A has an upper bound in A. (From non emptiness, it is sufficient to assume every pair of elements in A has an upper bound in A.) Also we say B is a lower set if $B = \downarrow B$. A subset J of P is called an ideal if it is directed lower set.

Definition 4 ([13]). Let P be a poset and $d\colon P \to P$ be a function. We call d a derivation on P, if it satisfies the following conditions:

(i) $d\big(l(x,y)\big) = l\Big(u\big(l(d(x),y), l(x,d(y))\big)\Big)$ for all $x, y \in P$;

(ii) $l\big(d(u(x,y))\big) = l\big(u(d(x),d(y))\big)$ for all $x, y \in P$.

Lemma 1 ([13]). *Let d be a derivation on P. Then the following hold:*

(i) $dx \le x$.

(ii) If $x \le y$, then $dx \le dy$.

(iii) If I is an ideal of P, then $dI \subseteq I$.

(iv) If P has the least element 0, then $d0 = 0$.

3 The generalized d-derivations of posets

Definition 5. Let P be a poset, $d\colon P \to P$ be a derivation and $F\colon P \to P$ be a function. We call F a generalized d-derivation on P, if it satisfies the following conditions:

(i) $F\big(l(x,y)\big) = l\Big(u\big(l(F(x),y), l(x,d(y))\big)\Big)$ for all $x, y \in P$;

(ii) $l\big(F(u(x,y))\big) = l\Big(u\big(F(x),F(y)\big)\Big)$ for all $x, y \in P$.

From the Remark 1 of [13], we can obtain the following remark:

Remark 1. Suppose $P = (P, \le, \wedge, \vee)$ is a lattice. It is easy to prove that if F is a generalized d-derivation on $(P, \le)$, then F is a generalized d-derivation on lattice $(P, \wedge, \vee)$.

Remark 2. Clearly if P is any poset, then every derivation d on P is a generalized d-derivation on P.

Now we give an example of a generalized d-derivation F on a poset P which is not a derivation on P.

Example 1. Let the poset $(P, \le) = ([0, \infty), \le)$. Define the functions $F, d\colon [0, \infty) \to [0, \infty)$ by $d(x) = 0$, $F(x) = \frac{1}{2}x$. Then d is a derivation and F is a generalized d-derivation on P which is not a derivation on P.

Note that, in the following P is a poset and $d\colon P \to P$ is a derivation.

Proposition 1. *Let F be a generalized d-derivation on P. Then the following hold:*

(i) If $x \le y$, then $F(x) \le F(y)$;

(ii) $d(x) \le F(x)$ for all $x \in P$;

(iii) $F(x) \le x$ for all $x \in P$;

(iv) If I is an ideal of P, then $F(I) \subseteq I$;

(v) *If P has the least element* 0, *then* $F(0) = 0$.

Proof. (i) Assume that $x \leq y$, then

$$l\big(F(u(x,y))\big) = l\big(F(u(y))\big) = l\big(u(F(x), F(y))\big).$$

But $F(x) \in l\big(u(F(x), F(y))\big)$, so $F(x) \in l(F(u(y)))$. Hence $F(x) \leq F(y)$.

(ii) Let $x \in P$ by Lemma 1, we get $d(x) \leq x$ and

$$l(d(x)) = l(x, d(x)) = l\big(u(l(x, d(x)))\big).$$

Using $l(u(l(x, d(x)))) \subseteq l(u(l(F(x), x), l(x, d(x))))$ and

$$l\Big(u\big(l(F(x), x), l(x, d(x))\big)\Big) = F\big(l(x,x)\big) = F(l(x))$$

and also the fact that $F(l(x)) \subseteq l(F(x))$, we obtain $d(x) \in l(F(x))$ which forces that $d(x) \leq F(x)$.

(iii) Since F is a generalized d-derivation on P, using (ii) we have

$$\begin{aligned} F(l(x,x)) &= l(u(l(F(x), x), l(x, d(x)))) = l(u(l(F(x), x), l(d(x)))) \\ &= l(u(l(F(x), x))) = l(F(x), x). \end{aligned}$$

Since

$$F(l(x)) = F(l(x,x)) = l(F(x), x)$$

and $F(x) \in F(l(x))$, we obtain $F(x) \in l(F(x), x)$. Hence $F(x) \leq x$.

(iv) Assume that I is an ideal of P. Let $x \in F(I)$, then there exists $y \in I$ such that $F(y) = x$. Using (iii), we get $F(y) \leq y$ and so $x \leq y$, but I is an ideal of P. Hence $x \in I$. So we proved $d(I) \subseteq I$. (v) Let P has the least element 0 then, by (iii), we get $0 \leq F(0) \leq 0$. Hence $F(0) = 0$.

$\square$

Note that, if we put $F = d$ in the Proposition 1, we obtain Lemma 1.

Lemma 2. *Let F be a generalized d-derivation on P. Then the following hold:*

(i) *if* $F(l(x)) = l(y)$, *then* $F(x) = y$.

(ii) *if* $F(u(x)) = u(y)$, *then* $F(x) = y$.

Proof. (i) Assume that $F(l(x)) = l(y)$. Since $y \in l(y)$, we get $y \in F(l(x))$, there exists $z \in l(x)$ such that $F(z) = y$. But $F(z) \leq F(x)$, so $y \leq F(x)$. On the other hand $F(x) \in F(l(x)) = l(y)$, so $F(x) \leq y$. Hence $F(x) = y$.

(ii) To avoid any repetitions, we use a similar demonstration to that in (i), we find the result requested.

$\square$

Note that in the previous Lemma if we put $F = d$ we obtain [13, Lemma 2.1].

Theorem 1. *Let P be a poset, $d\colon P \to P$ be a derivation and $F\colon P \to P$ is a function. Then F is a generalized d-derivation on P, if and only if the following conditions hold*

(i) $F(l(x,y)) = l(F(x),y) = l(x,F(y))$ *for all* $x,y \in P$*;*

(ii) $l(F(u(x,y))) = l(u(F(x),F(y)))$ *for all* $x,y \in P$*.*

Proof. We just need to demonstrate that the condition (i) in Definition 5 is equivalent to the one (i) in this Theorem.

First assume the condition (i) in this theorem is satisfied and $x,y \in P$. So

$$F(l(x,y)) = l(F(x),y) = l(u(l(F(x),y))),$$

and using

$$l\Big(u\big(l(F(x),y)\big)\Big) \subseteq l\Big(u\big(l(F(x),y),l(x,d(y))\big)\Big)\,,$$

then

$$F(l(x,y)) \subseteq l\Big(u\big(l(F(x),y),l(x,d(y))\big)\Big)\,.$$

On the other hand by Proposition 1 (ii), we get

$$l\Big(u\big(l(F(x),y),l(x,d(y))\big)\Big) \subseteq l\Big(u\big(l(F(x),y),l(x,F(y))\big)\Big) \quad \text{for all } x,y \in P$$

and

$$l\Big(u\big(l(F(x),y),l(x,F(y))\big)\Big) = l\big(u(l(F(x),y))\big) = l(F(x),y) = F(l(x,y)).$$

Thus

$$l\Big(u\big(l(F(x),y),l(x,d(y))\big)\Big) \subseteq F(l(x,y))\,,$$

finally the last two expressions give equality.

Second suppose F is a generalized d-derivation on P. Since

$$l(F(x),y) = l(u(l(F(x),y)))\,,$$
$$l(u(l(F(x),y),l(x,d(y)))) = F(l(x,y))$$

and

$$l(u(l(F(x),y))) \subseteq l(u(l(F(x),y),l(x,d(y))))\,,$$

we get

$$l(F(x),y) \subseteq l(u(l(F(x),y),l(x,d(y))))\,.$$

On the other hand, assume that $z \in F(l(x,y))$, then there exists $v \in l(x,y)$ satisfying $F(v) = z$. By Proposition 1 (i) and Proposition 1 (iii), we have $F(v) \leq F(x)$

and $F(v) \leq F(y) \leq y$. This shows that $z = F(v) \in l(F(x), y)$. Thus $F(l(x,y)) \subseteq l(F(x), y)$. Hence $F(l(x,y)) = l(F(x), y)$.

Since $F(l(x,y)) = l(F(x), y)$ so

$$F(l(y,x)) = l(F(y), x) = l(x, F(y)) .$$

But $F(l(x,y)) = F(l(y,x))$, then we get $l(F(x), y) = l(x, F(y))$. Hence

$$F(l(x,y)) = l(F(x), y) = l(x, F(y)) .$$

□

If we put $F = d$ in Theorem 1, we obtain [13, Theorem 2.2].

Corollary 1. *Every generalized d-derivation on P is a meet-translation.*

Proposition 2. *Let F be a generalized d-derivation on P. Then*

$$F(l(x,y)) = l(F(x), F(y))$$

for all $x, y \in P$.

Proof. Assume that $t \in l(F(x), F(y))$. Then $t \leq F(x)$ and $t \leq F(y)$. By Proposition 1 (i), we get $t \leq y$ which implies that $t \in l(F(x), y)$. By Theorem 1 (i), we obtain $l(F(x), F(y)) \subseteq F(l(x,y))$. To show the second inclusion, suppose $t \in F(l(x,y))$, so there exists $z \in l(x,y)$ such that $F(z) = t$. Thus by Proposition 1 (i), we get $F(z) \leq F(x)$, $F(z) \leq F(y)$. Thus $t \in l(F(x), F(y))$. Hence $F(l(x,y)) \subseteq l(F(x), F(y))$. □

If we put $F = d$ in Proposition 2, we obtain [13, Proposition 2.3].

Corollary 2. *Every generalized d-derivation on P is a lower homomorphism.*

4 The fixed points of generalized d-derivations

In this section P is a poset, $d \colon P \to P$ is a derivation and F is a generalized d-derivation on P. Denote $\mathrm{Fix}_F(P) = \{x \in P : F(x) = x\}$ and $F(P) = \{F(x) : x \in P\}$.

Theorem 2. *Let F be a generalized d-derivation on P. Then*

(i) $F(x) \in \mathrm{Fix}_F(P)$*, for all* $x \in P$*,*

(ii) $\mathrm{Fix}_F(P) = F(P)$*.*

Proof. (i) Let $x \in P$, from Proposition 1 (i) and Theorem 1 (i), we obtain

$$F(l(F(x))) = F(l(x, F(x))) = l(F(x), F(x)) = l(F(x))$$

and by Lemma 2 (i), we obtain $F(F(x)) = F(x)$. Hence $F(x) \in \mathrm{Fix}_F(P)$.

(ii) By (i), we have $F(x) \in \mathrm{Fix}_F(P)$, for all $x \in P$, then $F(P) \subseteq \mathrm{Fix}_F(P)$. On the other hand let $x \in \mathrm{Fix}_F(P)$, so $x = F(x) \in F(P)$. Thus $\mathrm{Fix}_F(P) \subseteq F(P)$, and hence $\mathrm{Fix}_F(P) = F(P)$.

□

We must notice that if we put $F = d$ in Theorem 2, we obtain [13, Theorem 3.1].

Proposition 3. *Let F, T be generalized d-derivations on P. Then $F = T$ if and only if $\mathrm{Fix}_F(P) = \mathrm{Fix}_T(P)$.*

Proof. It is clear that if $F = T$, then $\mathrm{Fix}_F(P) = \mathrm{Fix}_T(P)$. Conversely, let $\mathrm{Fix}_F(P) = \mathrm{Fix}_T(P)$, and $x \in P$. But by Theorem 2 (i) we get $F(x) \in \mathrm{Fix}_F(P) = \mathrm{Fix}_T(P)$, so $T(F(x)) = F(x)$. Similarly, we also have $F(T(x)) = T(x)$. So by Proposition 1 (i), (iii) we obtain $F(x) \leq T(x)$ and $T(x) \leq F(x)$. Thus $F(x) = T(x)$ and hence $F = T$. □

Proposition 4. *Let F be generalized d-derivation on P and P has a least element 0. Then the following hold.*

(i) $0 \in \mathrm{Fix}_F(P)$.

(ii) *If $x \in \mathrm{Fix}_F(P)$ and $y \leq x$, then $y \in \mathrm{Fix}_F(P)$.*

(iii) *If P is directed, then for any $x, y \in \mathrm{Fix}_F(P)$, there exists $z \in \mathrm{Fix}_F(P)$ satisfying $x \leq z$, $y \leq z$.*

Proof. (i) Since $F(0) = 0$, then $0 \in \mathrm{Fix}_F(P)$.

(ii) Assume that $x \in \mathrm{Fix}_F(P)$ and $y \leq x$, then $F(x) = x$. By Theorem 1 (i), we get

$$F(l(y)) = F(l(x, y)) = l(F(x), y) = l(x, y) = l(y).$$

Using Lemma 2 (i), we arrive at $F(y) = y$, and hence $y \in \mathrm{Fix}_F(P)$.

(iii) Assume that P is directed then for any $x, y \in P$ there exists $v \in P$ such that $x \leq v$ and $y \leq v$. Since $x, y \in \mathrm{Fix}_F$, then $F(x) = x$, $d(y) = y$. But $F(x) = x \leq F(v)$ and $F(y) = y \leq F(v)$. Put $z = F(v)$, hence by Theorem 2 (ii), we get $z \in \mathrm{Fix}_F(P)$. □

If we Put $F = d$ in the previous Proposition, we obtain [13, Proposition 3.3].

Corollary 3. *If P is a directed poset with the least element 0, then $\mathrm{Fix}_F(P)$ is an ideal of P.*

Theorem 3. *Let F be a generalized d-derivation on P. Then for all $x \in P$,*

$$\mathrm{Fix}_F(P) \cap l(x) = l(F(x))\,.$$

Proof. Suppose that $y \in \mathrm{Fix}_F(P) \cap l(x)$, then $F(y) = y$ and $y \leq x$. So $F(y) \leq F(x)$ and we have $y \leq F(x)$. Thus $y \in l(F(x))$, and hence

$$\mathrm{Fix}_F(P) \cap l(x) \subseteq l(F(x))\,.$$

On the other hand we have $F(x) \leq x$ and $F(x) \in \mathrm{Fix}_F(P)$. So

$$F(x) \in \mathrm{Fix}_F(P) \cap l(x)\,.$$

Thus we get $l(F(x)) \subseteq \mathrm{Fix}_F(P) \cap l(x)$. Hence $\mathrm{Fix}_F(P) \cap l(x) = l(F(x))$. □

If we Put $F = d$ in Theorem 3, we obtain [13, Theorem 3.4].

5 The ideals and operations related with generalized d-derivations

In this section, P is a poset with the least element 0 and $d\colon P \to P$ is a derivation.

Theorem 4. *Let F be a generalized d-derivation on P. Then*

$$\ker F = \{x \in P : F(x) = 0\}$$

is a nonempty lower set of P.

Proof. From Proposition 1 (v), we have $F(0) = 0$. So $0 \in \ker F$, and $\ker F \neq \emptyset$. Assume that $x \in \ker F$, and $y \leq x$ then $F(x) = 0$. By Proposition 1 (i), we get $F(y) \leq F(x) = 0$. Thus $F(y) = 0$, and this shows that $y \in \ker F$. □

We must notice that if we put $F = d$ in Theorem 4 we obtain [13, Theorem 4.1].

Proposition 5. *Let F be a generalized d-derivation on P, and I be an ideal of P. Then $F^{-1}(I)$ is a nonempty lower set of P such that $\ker F \subseteq F^{-1}(I)$.*

Proof. Since $F(0) = 0$, we have $0 \in F^{-1}(I)$ and then $F^{-1}(I) \neq \emptyset$. Suppose $x \in F^{-1}(I)$ and $y \leq x$, then $F(x) \in I$. By Proposition 1 (i), $F(y) \leq F(x) \in I$ which implies that $F(y) \in I$ and this shows that $y \in F^{-1}(I)$. Hence $F^{-1}(I)$ is a nonempty lower set of P. On the other hand, note that $\ker F = F^{-1}(\{0\}) \subseteq F^{-1}(I)$. □

If we put $F = d$ in Proposition 5, we obtain [13, Proposition 4.2].

Proposition 6. *Let F be a generalized d-derivation on P and I, J be two ideals of P. Then we have:*

(i) if $I \subseteq J$, then $F(I) \subseteq F(J)$,

(ii) $F(I \cap J) = F(I) \cap F(J)$.

Proof. (i) Assume that $x \in F(I)$, then there exist $y \in I \subseteq J$ such that $x = F(y)$. Hence $x \in F(J)$,and this shows that $F(I) \subseteq F(J)$.

(ii) It is clear that $F(I \cap J) \subseteq F(I) \cap F(J)$. Let $x \in F(I) \cap F(J)$ then there exist $a \in I$ and $b \in J$ satisfying $F(a) = x$, $F(b) = x$. By Theorem 1 (i), we get

$$F(l(a, F(b))) = l(F(a), F(b)) = l(x, x) = l(x).$$

But $x \in l(x)$, so we have $x \in F(l(a, F(b))$. Thus there exists $z \in l(a, F(b))$ satisfying $F(z) = x$. Also $z \leq a$, $z \leq F(b) \leq b$. This means $z \in I \cap J$, and hence $x \in F(I \cap J)$. Therefore $F(I) \cap F(J) \subseteq F(I \cap J)$.

□

Notice that, if we put $F = d$ in Proposition 6, we obtain Proposition 4.3 of [13].

We must notice that by the product dt of two functions d and t of a set P into itself, we mean, as usual, the function dt is defined by $dt(x) = d(t(x))$.

Theorem 5. *If F, T are generalized d-derivations on P, then F and T are commutated.*

Proof. Assume that F, T are generalized d-derivations on P. So, for any $x \in P$,

$$F(l(T(x))) = F(l(x, T(x))) = l(F(x), T(x))$$

and

$$T(l(F(x))) = T(l(x, F(x))) = l(T(x), F(x)).$$

Thus $F\big(l(T(x))\big) = T\big(l(F(x))\big)$. But

$$FT(x) = F(T(x)) \in F\big(l(T(x))\big)\,,$$

thus $FT(x) \in T\big(l(F(x))\big)$. Then there exists $y \in l(F(x))$ such that $FT(x) = T(y)$. By Proposition 1 (i), $T(y) \leq T(F(x))$, and therefore $FT(x) \leq TF(x)$. Similarly, we can prove that $TF(x) \leq FT(x)$. This means $FT(x) = TF(x)$. $\square$

Theorem 6. *If F, T are generalized d-derivations on P, then $F \leq T$ if and only if $TF = F$.*

Proof. Let F, T are generalized d-derivations on P with $F \leq T$. So, for any $x \in P$, we have $F(x) \in \mathrm{Fix}_F(P)$ i.e. $F(x) = F(F(x)) \leq T(F(x))$. Also by Proposition 1 (iii), we get $TF(x) \leq F(x)$. Thus $TF(x) = F(x)$. This shows that $TF = F$. Conversely, for any $x \in P$, $F(x) = TF(x) \leq T(x)$. This means $F \leq T$. $\square$

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